EXERCISE 6.1 - SEQUENCES AND SERIES

Complete Solutions

Question 1: Find the next four terms of each sequence

(i) 12, 16, 20, ...

Step 1: Identify the pattern

- 12 → 16 (add 4)
- 16 → 20 (add 4)
- This is an arithmetic sequence with common difference $d = 4$

Step 2: Find the next four terms

- 4th term: $20 + 4 = 24$
- 5th term: $24 + 4 = 28$
- 6th term: $28 + 4 = 32$
- 7th term: $32 + 4 = 36$

Answer: 24, 28, 32, 36

(ii) 3, 1, -1, ...

Step 1: Identify the pattern

- 3 → 1 (subtract 2)
- 1 → -1 (subtract 2)
- This is an arithmetic sequence with common difference $d = -2$

Step 2: Find the next four terms

- 4th term: $-1 + (-2) = -3$
- 5th term: $-3 + (-2) = -5$
- 6th term: $-5 + (-2) = -7$
- 7th term: $-7 + (-2) = -9$

Answer: -3, -5, -7, -9

Question 2: Write down the first three terms of each sequence

(i) $a_n = 3n + 5$

Step 1: Find a_1, a_2, a_3

- $a_1 = 3(1) + 5 = 3 + 5 = 8$
- $a_2 = 3(2) + 5 = 6 + 5 = 11$
- $a_3 = 3(3) + 5 = 9 + 5 = 14$

Answer: 8, 11, 14

(ii) $a_{n+1} = 4a_n - 7$ and $a_1 = 3$

Step 1: Find a_2, a_3, a_4

- $a_1 = 3$ (given)

- $a_2 = 4(3) - 7 = 12 - 7 = 5$
- $a_3 = 4(5) - 7 = 20 - 7 = 13$
- $a_4 = 4(13) - 7 = 52 - 7 = 45$

Answer: 3, 5, 13 (first three terms)

(iii) $\mathbf{a_n = (n-3)(n+1)}$

Step 1: Find a_1, a_2, a_3

- $a_1 = (1-3)(1+1) = (-2)(2) = -4$
- $a_2 = (2-3)(2+1) = (-1)(3) = -3$
- $a_3 = (3-3)(3+1) = (0)(4) = 0$

Answer: -4, -3, 0

(iv) $\mathbf{a_1 = -1,\ a_{n+1} = \frac{3}{a_n+2}}$

Step 1: Find a_2, a_3, a_4

- $a_1 = -1$ (given)
- $a_2 = \frac{3}{-1+2} = \frac{3}{1} = 3$
- $a_3 = \frac{3}{3+2} = \frac{3}{5}$
- $a_4 = \frac{3}{\frac{3}{5}+2} = \frac{3}{\frac{3}{5}+\frac{10}{5}} = \frac{3}{\frac{13}{5}} = \frac{15}{13}$

Answer: -1, 3, $\frac{3}{5}$ (first three terms)

(v) $\mathbf{a_n = 8 - \frac{20}{3+n}}$

Step 1: Find a_1, a_2, a_3

- $a_1 = 8 - \frac{20}{3+1} = 8 - \frac{20}{4} = 8 - 5 = 3$
- $a_2 = 8 - \frac{20}{3+2} = 8 - \frac{20}{5} = 8 - 4 = 4$
- $a_3 = 8 - \frac{20}{3+3} = 8 - \frac{20}{6} = 8 - \frac{10}{3} = \frac{24}{3} - \frac{10}{3} = \frac{14}{3}$

Answer: 3, 4, $\frac{14}{3}$

(vi) $\mathbf{a_1 = 1,\ a_{n+1} = (3a_n+2)^2}$

Step 1: Find a_2, a_3, a_4

- $a_1 = 1$ (given)
- $a_2 = (3(1)+2)^2 = (3+2)^2 = 5^2 = 25$
- $a_3 = (3(25)+2)^2 = (75+2)^2 = 77^2 = 5929$
- $a_4 = (3(5929)+2)^2 = (17787+2)^2 = 17789^2 = 316,448,521$

Answer: 1, 25, 5929 (first three terms)

(vii) $\mathbf{a_n = (-2)^n}$

Step 1: Find a_1, a_2, a_3

- $a_1 = (-2)^1 = -2$

- $a_2 = (-2)^2 = 4$
- $a_3 = (-2)^3 = -8$

Answer: -2, 4, -8

(viii) $\mathbf{a_n = (-1)^n 7^n}$

Step 1: Find a_1, a_2, a_3

- $a_1 = (-1)^1 \cdot 7^1 = -1 \cdot 7 = -7$
- $a_2 = (-1)^2 \cdot 7^2 = 1 \cdot 49 = 49$
- $a_3 = (-1)^3 \cdot 7^3 = -1 \cdot 343 = -343$

Answer: -7, 49, -343

Question 3: Triangular Numbers

Given: $\mathbf{T_n = \frac{n(n+1)}{2}}$

(i) Find the 15th triangular number

Step 1: Substitute $n = 15$ into the formula

$$T_{15} = \frac{15(15+1)}{2} = \frac{15 \times 16}{2} = \frac{240}{2} = 120$$

Answer: The 15th triangular number is **120**

(ii) Make a triangle of dots for n = 5

Step 1: For $n = 5$, we need 5 rows of dots

Step 2: Row pattern:

- Row 1: 1 dot
- Row 2: 2 dots
- Row 3: 3 dots
- Row 4: 4 dots
- Row 5: 5 dots

Step 3: Draw the triangle

text

Step 4: Verify total number of dots

$$T_5 = \frac{5(5+1)}{2} = \frac{5 \times 6}{2} = 15$$

Counting the dots in the triangle:
1 + 2 + 3 + 4 + 5 = 15

Summary Table

1(i)	24, 28, 32, 36
1(ii)	-3, -5, -7, -9
2(i)	8, 11, 14
2(ii)	3, 5, 13
2(iii)	-4, -3, 0
2(iv)	-1, 3, $\frac{3}{5}$
2(v)	3, 4, $\frac{14}{3}$
2(vi)	1, 25, 5929
2(vii)	-2, 4, -8
2(viii)	-7, 49, -343
3(i)	120
3(ii)	Triangle of dots shown above

EXERCISE 6.1 - SEQUENCES (Continued)

Question 4: Write down the n^{th} term of each sequence

(i) $1, 4, 9, \ldots$

Step 1: Observe the pattern

- $a_1 = 1 = 1^2$
- $a_2 = 4 = 2^2$
- $a_3 = 9 = 3^2$

Step 2: General formula

- The terms are squares of natural numbers
- $a_n = n^2$

Answer: $a_n = n^2$

(ii) $1, 1+2, 1+2+3, \ldots$

Step 1: Observe the pattern

- $a_1 = 1$
- $a_2 = 1 + 2 = 3$
- $a_3 = 1 + 2 + 3 = 6$

Step 2: Recognize the pattern

- These are triangular numbers
- $a_n = 1 + 2 + 3 + \cdots + n$

Step 3: Use the formula for sum of first n natural numbers

- $a_n = \frac{n(n+1)}{2}$

Answer: $a_n = \frac{n(n+1)}{2}$

(iii) $\mathbf{a_1, b_1, a_2, b_2, a_3, b_3, \ldots}$

Step 1: Observe the pattern

- Odd terms: $a_1, a_2, a_3, \ldots$
- Even terms: $b_1, b_2, b_3, \ldots$

Step 2: Express using piecewise function

For odd n: $n = 2k - 1$, term is a_k
For even n: $n = 2k$, term is b_k

Answer:

$$a_n = \begin{cases} a_{\frac{n+1}{2}} & \text{if } n \text{ is odd} \\ b_{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$$

(iv) $\mathbf{x, 2x^2, 3x^3, \ldots}$

Step 1: Observe the pattern

- $a_1 = 1 \cdot x^1 = x$
- $a_2 = 2 \cdot x^2 = 2x^2$
- $a_3 = 3 \cdot x^3 = 3x^3$

Step 2: General formula

- Coefficient: n
- Power of x: n
- $a_n = n \cdot x^n$

Answer: $a_n = nx^n$

(v) $\mathbf{a_1, a_1+d, a_1+2d, \ldots}$

Step 1: Observe the pattern

- $a_1 = a_1 + 0d$
- $a_2 = a_1 + 1d$
- $a_3 = a_1 + 2d$

Step 2: General formula

- This is an arithmetic sequence
- First term: a_1
- Common difference: d

Answer: $a_n = a_1 + (n-1)d$

(vi) $\mathbf{a_1, a_1 r, a_1 r^2, \ldots}$

Step 1: Observe the pattern

- $a_1 = a_1 \cdot r^0$

- $a_2 = a_1 \cdot r^1$
- $a_3 = a_1 \cdot r^2$

Step 2: General formula

- This is a geometric sequence
- First term: a_1
- Common ratio: r

Answer: $a_n = a_1 r^{n-1}$

(vii) $\frac{a_1}{b_1+c_1}, \frac{2a_2}{b_2+c_2}, \frac{3a_3}{b_3+c_3}, \ldots$

Step 1: Observe the pattern for each term n

- Numerator: $n \cdot a_n$
- Denominator: $b_n + c_n$

Step 2: General formula

Answer: $a_n = \frac{na_n}{b_n+c_n}$

(viii) $\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \ldots$

Step 1: Observe the pattern

- $a_1 = \frac{1}{a_1}$
- $a_2 = \frac{1}{a_1+d}$
- $a_3 = \frac{1}{a_1+2d}$

Step 2: General formula

- Denominator follows arithmetic sequence: $a_1 + (n-1)d$

Answer: $a_n = \frac{1}{a_1+(n-1)d}$

Summary Table

Part	Sequence	n^{th} Term
(i)	$1, 4, 9, \ldots$	$a_n = n^2$
(ii)	$1, 1+2, 1+2+3, \ldots$	$a_n = \frac{n(n+1)}{2}$
(iii)	$a_1, b_1, a_2, b_2, a_3, b_3, \ldots$	Piecewise as shown
(iv)	$x, 2x^2, 3x^3, \ldots$	$a_n = nx^n$
(v)	$a_1, a_1+d, a_1+2d, \ldots$	$a_n = a_1 + (n-1)d$
(vi)	$a_1, a_1r, a_1r^2, \ldots$	$a_n = a_1r^{n-1}$
(vii)	$\frac{a_1}{b_1+c_1}, \frac{2a_2}{b_2+c_2}, \ldots$	$a_n = \frac{na_n}{b_n+c_n}$
(viii)	$\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \ldots$	$a_n = \frac{1}{a_1+(n-1)d}$