## EXERCISE 6.3 - ARITHMETIC MEANS

**Complete Solutions**

### Question 1: Find A.M. between the given numbers

#### (i) $2+\sqrt{3}i, 2-\sqrt{3}i$

**Step 1:** Arithmetic Mean formula

$$\text{A.M.} = \frac{\text{First} + \text{Second}}{2}$$

**Step 2:** Substitute

$$\text{A.M.} = \frac{(2+\sqrt{3}i)+(2-\sqrt{3}i)}{2} = \frac{4}{2} = 2$$

**Answer:** A.M. $= 2$

#### (ii) $(a+b)^2, (a-b)^2$

**Step 1:** Arithmetic Mean formula

$$\text{A.M.} = \frac{(a+b)^2+(a-b)^2}{2}$$

**Step 2:** Expand

$$= \frac{(a^2+2ab+b^2)+(a^2-2ab+b^2)}{2} = \frac{2a^2+2b^2}{2} = a^2+b^2$$

**Answer:** A.M. $= a^2+b^2$

### Question 2: If 6, 11, 16 are three A.Ms. between a and b

**Step 1:** The sequence is: $a, 6, 11, 16, b$

- This is an A.P. with 5 terms

**Step 2:** Find common difference

- 6, 11, 16 have common difference $d = 5$

**Step 3:** Find $a$

- $a + 5 = 6 \rightarrow a = 1$

**Step 4:** Find $b$

- $16 + 5 = b \rightarrow b = 21$

**Answer:** $a = 1, b = 21$

### Question 3: Insert five A.Ms. between $\sqrt{2}$ and $\frac{15}{\sqrt{2}}$

**Step 1:** We need sequence: $\sqrt{2}, \text{A}_1, \text{A}_2, \text{A}_3, \text{A}_4, \text{A}_5, \frac{15}{\sqrt{2}}$

- Total 7 terms in A.P.

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**Step 2:** First term $a = \sqrt{2}$, last term $b = \frac{15}{\sqrt{2}}$, number of terms $n = 7$

**Step 3:** Use formula $b = a + (n-1)d$

$$\frac{15}{\sqrt{2}} = \sqrt{2} + 6d \frac{15}{\sqrt{2}} - \sqrt{2} = 6d \frac{15-2}{\sqrt{2}} = 6d \frac{13}{\sqrt{2}} = 6dd = \frac{13}{6\sqrt{2}}$$

**Step 4:** Find the arithmetic means

- $A_1 = \sqrt{2} + d = \sqrt{2} + \frac{13}{6\sqrt{2}} = \frac{6(2)+13}{6\sqrt{2}} = \frac{25}{6\sqrt{2}}$
- $A_2 = \sqrt{2} + 2d = \sqrt{2} + \frac{13}{3\sqrt{2}} = \frac{6+26}{3\sqrt{2}} = \frac{32}{3\sqrt{2}}$
- $A_3 = \sqrt{2} + 3d = \sqrt{2} + \frac{13}{2\sqrt{2}} = \frac{4+13}{2\sqrt{2}} = \frac{17}{2\sqrt{2}}$
- $A_4 = \sqrt{2} + 4d = \sqrt{2} + \frac{26}{3\sqrt{2}} = \frac{6+26}{3\sqrt{2}} = \frac{32}{3\sqrt{2}}$
- $A_5 = \sqrt{2} + 5d = \sqrt{2} + \frac{65}{6\sqrt{2}} = \frac{12+65}{6\sqrt{2}} = \frac{77}{6\sqrt{2}}$

**Answer:** $\frac{25}{6\sqrt{2}}, \frac{32}{3\sqrt{2}}, \frac{17}{2\sqrt{2}}, \frac{32}{3\sqrt{2}}, \frac{77}{6\sqrt{2}}$

## Question 4: A.M. of two numbers is 7 and product is 45

**Step 1:** Let the numbers be $x$ and $y$

- A.M. $= \frac{x+y}{2} = 7 \rightarrow x + y = 14$
- Product: $xy = 45$

**Step 2:** Solve the system

- $x + y = 14$ ... (1)
- $xy = 45$ ... (2)

**Step 3:** From (1): $y = 14 - x$
Substitute in (2): $x(14 - x) = 45$

$$14x - x^2 = 45x^2 - 14x + 45 = 0$$

**Step 4:** Factor

$$(x-5)(x-9) = 0x = 5$$

or $x = 9$

**Step 5:** If $x = 5, y = 9$; if $x = 9, y = 5$

**Answer:** The numbers are 5 and 9

## Question 5: If n arithmetic means are inserted between a and b

**Step 1:** Sequence: $a, A_1, A_2, \ldots, A_n, b$

- Total terms: $n + 2$

**Step 2:** Let common difference $= d$

- $b = a + (n+1)d$
- $(n+1)d = b - a$
- $d = \frac{b-a}{n+1}$

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**Answer:** $d = \frac{b-a}{n+1}$

### Question 6: If A is the A.M. between a and b

**Step 1:** Since $A$ is the A.M., $A = \frac{a+b}{2}$

**Step 2:** LHS $= (a-A)^2 + (A-b)^2$

**Step 3:** Substitute $A = \frac{a+b}{2}$

- $a - A = a - \frac{a+b}{2} = \frac{2a-a-b}{2} = \frac{a-b}{2}$
- $A - b = \frac{a+b}{2} - b = \frac{a+b-2b}{2} = \frac{a-b}{2}$

**Step 4:** LHS

$$\left(\frac{a-b}{2}\right)^2 + \left(\frac{a-b}{2}\right)^2 = \frac{(a-b)^2}{4} + \frac{(a-b)^2}{4} = \frac{(a-b)^2}{2}$$

**Therefore,** $(a-A)^2 + (A-b)^2 = \frac{1}{2}(a-b)^2$

### Question 7: For what value of n, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. between a and b?

**Step 1:** A.M. between $a$ and $b$ is $\frac{a+b}{2}$

**Step 2:** Set up equation

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$$

**Step 3:** Cross multiply

$$2(a^{n+1}+b^{n+1}) = (a+b)(a^n+b^n) 2a^{n+1} + 2b^{n+1} = a^{n+1} + ab^n + a^nb + b^{n+1} a^{n+1} + b^{n+1} = ab^n + a^nb$$

**Step 4:** Rearrange

$$a^{n+1} - a^nb + b^{n+1} - ab^n = 0 a^n(a-b) + b^n(b-a) = 0 a^n(a-b) - b^n(a-b) = 0 (a-b)(a^n-b^n) = 0$$

**Step 5:** Since $a \neq b$, we have

$$a^n - b^n = 0 a^n = b^n$$

**Step 6:** For $a \neq b$, this holds when $n = 0$

Check: For $n = 0$:

$\frac{a^{0+1}+b^{0+1}}{a^0+b^0} = \frac{a+b}{1+1} = \frac{a+b}{2}$

**Answer:** $n = 0$

**Summary Table**

| | |
|---|---|
| 1(i) | A.M. $= 2$ |
| 1(ii) | A.M. $= a^2 + b^2$ |
| 2 | $a = 1, b = 21$ |
| 3 | $\frac{25}{6\sqrt{2}}, \frac{32}{3\sqrt{2}}, \frac{17}{2\sqrt{2}}, \frac{32}{3\sqrt{2}}, \frac{77}{6\sqrt{2}}$ |
| 4 | 5 and 9 |
| 5 | $d = \frac{b-a}{n+1}$ |
| 6 | Identity proven |
| 7 | $n = 0$ |

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