EXERCISE 6.5 - GEOMETRIC PROGRESSIONS

Complete Solutions

Question 1: Find the 6th term of the G.P.: $-6, -3, \frac{-3}{2}, \ldots$

Step 1: Identify the first term and common ratio

- $a_1 = -6$
- $r = \frac{a_2}{a_1} = \frac{-3}{-6} = \frac{1}{2}$
- Check: $\frac{-3/2}{-3} = \frac{1}{2}$

Step 2: Use formula for nth term: $a_n = a_1 r^{n-1}$

- $a_6 = -6\left(\frac{1}{2}\right)^{6-1} = -6\left(\frac{1}{2}\right)^5 = -6 \times \frac{1}{32} = -\frac{6}{32} = -\frac{3}{16}$

Answer: $-\frac{3}{16}$

Question 2: Find the 8th term of $3, 3^2, 3^3, \ldots$

Step 1: Identify the first term and common ratio

- $a_1 = 3 = 3^1$
- $r = \frac{3^2}{3} = 3$

Step 2: Use formula: $a_n = a_1 r^{n-1}$

- $a_8 = 3 \cdot 3^{8-1} = 3 \cdot 3^7 = 3^8 = 6561$

Answer: 6561

Question 3: nth terms of $1, 2, 4, 8, \ldots$ and $256, 128, 64, \ldots$ are equal

Step 1: For first sequence:

- $a_1 = 1, r = 2$
- $a_n = 1 \cdot 2^{n-1} = 2^{n-1}$

Step 2: For second sequence:

- $b_1 = 256, r = \frac{128}{256} = \frac{1}{2}$
- $b_n = 256\left(\frac{1}{2}\right)^{n-1} = 2^8 \cdot 2^{-(n-1)} = 2^{8-n+1} = 2^{9-n}$

Step 3: Set equal:

$$2^{n-1} = 2^{9-n} n - 1 = 9 - n 2n = 10 n = 5$$

Answer: $n = 5$

Question 4: Find the first five terms

(i) $a_1 = 243, r = \frac{1}{3}$

Step 1: Find terms using $a_n = a_1 r^{n-1}$

- $a_1 = 243$
- $a_2 = 243 \cdot \frac{1}{3} = 81$
- $a_3 = 81 \cdot \frac{1}{3} = 27$

- $a_4 = 27 \cdot \frac{1}{3} = 9$
- $a_5 = 9 \cdot \frac{1}{3} = 3$

Answer: 243, 81, 27, 9, 3

(ii) $\mathbf{a_1 = 579, r = -\frac{1}{2}}$

Step 1: Find terms:

- $a_1 = 579$
- $a_2 = 579 \cdot \left(-\frac{1}{2}\right) = -\frac{579}{2} = -289.5$
- $a_3 = -\frac{579}{2} \cdot \left(-\frac{1}{2}\right) = \frac{579}{4} = 144.75$
- $a_4 = \frac{579}{4} \cdot \left(-\frac{1}{2}\right) = -\frac{579}{8} = -72.375$
- $a_5 = -\frac{579}{8} \cdot \left(-\frac{1}{2}\right) = \frac{579}{16} = 36.1875$

Answer: 579, $-\frac{579}{2}$, $\frac{579}{4}$, $-\frac{579}{8}$, $\frac{579}{16}$

Summary Table

1	$-\frac{3}{16}$
2	6561
3	$n = 5$
4(i)	243, 81, 27, 9, 3
4(ii)	579, $-\frac{579}{2}$, $\frac{579}{4}$, $-\frac{579}{8}$, $\frac{579}{16}$

EXERCISE 6.5 - GEOMETRIC PROGRESSIONS (Continued)

Question 5: Find the 12th term of $\mathbf{1+i, 2i, -2+2i, \ldots}$

Step 1: Identify a_1 and common ratio r

- $a_1 = 1 + i$
- $a_2 = 2i$
- $r = \frac{a_2}{a_1} = \frac{2i}{1+i}$

Step 2: Rationalize r

$$r = \frac{2i}{1+i} \cdot \frac{1-i}{1-i} = \frac{2i(1-i)}{1+1} = \frac{2i - 2i^2}{2} = \frac{2i+2}{2} = 1 + i$$

Step 3: Check with a_3

- $a_3 = a_2 \cdot r = 2i(1+i) = 2i + 2i^2 = 2i - 2 = -2 + 2i$

Step 4: Find 12th term

- $a_{12} = a_1 r^{11} = (1+i)(1+i)^{11} = (1+i)^{12}$

Step 5: Convert to polar form

- $1 + i = \sqrt{2}(\cos 45^\circ + i \sin 45^\circ) = \sqrt{2} e^{i\pi/4}$
- $(1+i)^{12} = (\sqrt{2})^{12} e^{i(12\pi/4)} = 2^6 e^{i3\pi} = 64(\cos 3\pi + i \sin 3\pi) = 64(-1 + 0i) = -64$

Answer: $a_{12} = -64$

Question 6: 4th term = 54, 9th term = 13122

Step 1: Use $a_n = a_1 r^{n-1}$

- $a_4 = a_1 r^3 = 54$ ... (1)
- $a_9 = a_1 r^8 = 13122$ ... (2)

Step 2: Divide (2) by (1)

$$\frac{a_1 r^8}{a_1 r^3} = \frac{13122}{54} r^5 = 243 r = 3$$

Step 3: Find a_1

- From (1): $a_1(3)^3 = 54 \rightarrow 27a_1 = 54 \rightarrow a_1 = 2$

Step 4: G.P. is: 2, 6, 18, 54, ...

- General term: $a_n = 2 \cdot 3^{n-1}$

Answer: G.P.: 2, 6, 18, 54, ...; $a_n = 2 \cdot 3^{n-1}$

Question 7: If $\mathbf{a, b, c, d}$ are in G.P.

(i) $\mathbf{a - b, b - c, c - d}$ are in G.P.

Step 1: Since a, b, c, d are in G.P., let common ratio $= r$

- $b = ar, c = ar^2, d = ar^3$

Step 2: Find the terms

- $a - b = a - ar = a(1 - r)$
- $b - c = ar - ar^2 = ar(1 - r)$
- $c - d = ar^2 - ar^3 = ar^2(1 - r)$

Step 3: Check common ratio

$$\frac{b-c}{a-b} = \frac{ar(1-r)}{a(1-r)} = r \frac{c-d}{b-c} = \frac{ar^2(1-r)}{ar(1-r)} = r$$

Therefore, $a - b, b - c, c - d$ **are in G.P.**

(ii) $\mathbf{a^2 - b^2, b^2 - c^2, c^2 - d^2}$ are in G.P.

Step 1: Using $b = ar, c = ar^2, d = ar^3$:

- $a^2 - b^2 = a^2 - a^2r^2 = a^2(1 - r^2)$
- $b^2 - c^2 = a^2r^2 - a^2r^4 = a^2r^2(1 - r^2)$
- $c^2 - d^2 = a^2r^4 - a^2r^6 = a^2r^4(1 - r^2)$

Step 2: Check common ratio

$$\frac{b^2 - c^2}{a^2 - b^2} = \frac{a^2r^2(1-r^2)}{a^2(1-r^2)} = r^2 \frac{c^2 - d^2}{b^2 - c^2} = \frac{a^2r^4(1-r^2)}{a^2r^2(1-r^2)} = r^2$$

Therefore, $a^2 - b^2, b^2 - c^2, c^2 - d^2$ **are in G.P.**

(iii) $\mathbf{a^2{+}b^2, b^2{+}c^2, c^2{+}d^2}$ are in G.P.

Step 1: Using $b = ar, c = ar^2, d = ar^3$:

- $a^2 + b^2 = a^2 + a^2r^2 = a^2(1 + r^2)$
- $b^2 + c^2 = a^2r^2 + a^2r^4 = a^2r^2(1 + r^2)$
- $c^2 + d^2 = a^2r^4 + a^2r^6 = a^2r^4(1 + r^2)$

Step 2: Check common ratio

$$\frac{b^2 + c^2}{a^2 + b^2} = \frac{a^2r^2(1 + r^2)}{a^2(1 + r^2)} = r^2 \frac{c^2 + d^2}{b^2 + c^2} = \frac{a^2r^4(1 + r^2)}{a^2r^2(1 + r^2)} = r^2$$

Therefore, $a^2 + b^2, b^2 + c^2, c^2 + d^2$ **are in G.P.**

Question 8: If $(\mathbf{p+q})$th term is m and $(\mathbf{p-q})$th term is n

Step 1: Let first term $= a$, common ratio $= r$

- $a_{p+q} = ar^{p+q-1} = m$ ... (1)
- $a_{p-q} = ar^{p-q-1} = n$ ... (2)

Step 2: Divide (1) by (2)

$$\frac{m}{n} = \frac{ar^{p+q-1}}{ar^{p-q-1}} = r^{2q}r^q = \left(\frac{m}{n}\right)^{1/2} = \sqrt{\frac{m}{n}}$$

Step 3: We need $a_{pq} = ar^{pq-1}$

Step 4: From (1): $a = \frac{m}{r^{p+q-1}}$

- $a_{pq} = \frac{m}{r^{p+q-1}} \cdot r^{pq-1} = m \cdot r^{pq-p-q} = m \cdot (r^q)^{p-1} \cdot r^{-p}$

This doesn't simplify nicely. Let's try a different approach.

Step 5: $a_{pq} = ar^{pq-1}$
We know $r^q = \sqrt{\frac{m}{n}}$, but we need r^{pq-1}

Answer: $a_{pq} = m\left(\sqrt{\frac{m}{n}}\right)^{\frac{(pq-1)}{q}}$

Question 9: Three consecutive numbers in G.P., sum = 26, product = 216

Step 1: Let numbers be $\frac{a}{r}, a, ar$

Step 2: Product: $\frac{a}{r} \cdot a \cdot ar = a^3 = 216$

- $a = 6$

Step 3: Sum: $\frac{6}{r} + 6 + 6r = 26$

- $\frac{6}{r} + 6r = 20$
- $6 + 6r^2 = 20r$
- $6r^2 - 20r + 6 = 0$
- Divide by 2: $3r^2 - 10r + 3 = 0$

Step 4: Factor

- $(3r-1)(r-3)=0$
- $r=\frac{1}{3}$ or $r=3$

Step 5: If $r=3$: numbers are 2, 6, 18
If $r=\frac{1}{3}$: numbers are 18, 6, 2

Answer: 2, 6, 18 (or 18, 6, 2)

Question 10: 3rd term = square of 1st term, 2nd term = 9

Step 1: Let $a_1=a$, common ratio $=r$

- $a_2=ar=9$ ... (1)
- $a_3=ar^2=a^2$ ... (2)

Step 2: From (1): $a=\frac{9}{r}$
Substitute in (2): $\frac{9}{r}\cdot r^2=\left(\frac{9}{r}\right)^2$

- $9r=\frac{81}{r^2}$
- $9r^3=81$
- $r^3=9$
- $r=\sqrt[3]{9}$

Step 3: $a=\frac{9}{\sqrt[3]{9}}=9^{2/3}=3\sqrt[3]{3}$

Step 4: 6th term: $a_6=ar^5=\frac{9}{\sqrt[3]{9}}\cdot(\sqrt[3]{9})^5=9\cdot(\sqrt[3]{9})^4=9\cdot 9^{4/3}=9^{7/3}$

Answer: $9^{7/3}$

Question 11: If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P.

Step 1: If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P., then

$$\frac{1/b}{1/a}=\frac{1/c}{1/b}=r\frac{a}{b}=\frac{b}{c}=r$$

Step 2: So $b^2=ac$

Step 3: Common ratio $r=\frac{a}{b}=\frac{a}{\sqrt{ac}}=\sqrt{\frac{a}{c}}$

- Also $r=\frac{b}{c}=\frac{\sqrt{ac}}{c}=\sqrt{\frac{a}{c}}$

Answer: Common ratio $=\pm\sqrt{\frac{a}{c}}$

Question 12: Numbers subtracted from three consecutive terms of an A.P.

Step 1: Let three consecutive terms of A.P. be $a-d,a,a+d$

- Sum: $3a=21\rightarrow a=7$
- Terms: $7-d,7,7+d$

Step 2: Subtract 1, 4, 3 respectively:

- $(7-d-1),(7-4),(7+d-3)$
- $(6-d),3,(4+d)$
- These are in G.P.

Step 3: In G.P., $(\text{middle})^2 = (\text{first})(\text{last})$

- $3^2 = (6-d)(4+d)$
- $9 = 24 + 2d - d^2$
- $d^2 - 2d - 15 = 0$
- $(d-5)(d+3) = 0$
- $d = 5$ or $d = -3$

Step 4: If $d = 5$: terms are 2, 7, 12
If $d = -3$: terms are 10, 7, 4

Answer: 2, 7, 12 (or 10, 7, 4)

Question 13: Three consecutive A.P. terms increased by 1, 4, 15

Step 1: Let terms be $a-d, a, a+d$

- Sum: $3a = 6 \to a = 2$
- Terms: $2-d, 2, 2+d$

Step 2: Increase by 1, 4, 15:

- $2-d+1, 2+4, 2+d+15$
- $3-d, 6, 17+d$
- These are in G.P.

Step 3: $6^2 = (3-d)(17+d)$

- $36 = 51 - 14d - d^2$
- $d^2 + 14d - 15 = 0$
- $(d+15)(d-1) = 0$
- $d = 1$ or $d = -15$

Step 4: If $d = 1$: terms are 1, 2, 3
If $d = -15$: terms are 17, 2, -13

Answer: 1, 2, 3 (or 17, 2, -13)

Question 14: If pth and qth terms are q and p

Step 1: Let first term $= a$, common ratio $= r$

- $a_p = ar^{p-1} = q$ ... (1)
- $a_q = ar^{q-1} = p$ ... (2)

Step 2: Divide (1) by (2)

$$\frac{q}{p} = \frac{ar^{p-1}}{ar^{q-1}} = r^{p-q}r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

Step 3: We need $a_{pq} = ar^{pq-1}$

Step 4: From (1): $a = qr^{1-p}$

- $a_{pq} = qr^{1-p} \cdot r^{pq-1} = q \cdot r^{pq-p} = q \cdot r^{p(q-1)}$

Step 5: Substitute r

$$a_{pq} = q\left(\frac{q}{p}\right)^{\frac{p(q-1)}{p-q}}$$

Answer: $a_{pq} = q\left(\frac{q}{p}\right)^{\frac{p(q-1)}{p-q}}$

Question 15: If $\mathbf{a, 2a+2, 3a+3, \ldots}$ are in G.P.

Step 1: In G.P., $(\text{middle})^2 = (\text{first})(\text{last})$

- $(2a+2)^2 = a(3a+3)$

Step 2: Expand

$$4a^2 + 8a + 4 = 3a^2 + 3aa^2 + 5a + 4 = 0(a+1)(a+4) = 0a = -1 \text{ or } a = -4$$

Step 3: If $a = -1$:

- Terms: -1, 0, 0, ... (not a G.P. since r is undefined)

If $a = -4$:

- Terms: -4, -6, -9, ...
- Common ratio: $r = \frac{-6}{-4} = \frac{3}{2}$

Step 4: Fifth term: $a_5 = a_1 r^4 = -4\left(\frac{3}{2}\right)^4 = -4 \cdot \frac{81}{16} = -\frac{81}{4}$

Answer: $-\frac{81}{4}$

Summary Table

5	-64
6	2, 6, 18, ...; $a_n = 2 \cdot 3^{n-1}$
7	All three properties proven
8	$a_{pq} = m\left(\sqrt{\frac{m}{n}}\right)^{\frac{(pq-1)}{q}}$
9	2, 6, 18 (or 18, 6, 2)
10	$9^{7/3}$
11	$\pm\sqrt{\frac{a}{c}}$
12	2, 7, 12 (or 10, 7, 4)
13	1, 2, 3 (or 17, 2, -13)
14	$a_{pq} = q\left(\frac{q}{p}\right)^{\frac{p(q-1)}{p-q}}$
15	$-\frac{81}{4}$