EXERCISE 7.1 - PERMUTATIONS AND COMBINATIONS

Complete Solutions

Question 1: Evaluate each of the following

(i) $\frac{101}{0!\,8!}$

Step 1: Note: $0! = 1$

Step 2: $\frac{101}{1\times 8!} = \frac{101}{8!} = \frac{101}{40320}$

Answer: $\frac{101}{40320}$

(ii) $\frac{121}{3!(12-3)!}$

Step 1: Simplify denominator

$$3!(12-3)! = 3! \cdot 9!$$

Step 2: $\frac{121}{3! \cdot 9!} = \frac{121}{6\times 362880} = \frac{121}{2177280}$

Answer: $\frac{121}{2177280}$

(iii) $\frac{1440}{3!\,4!} + \frac{2400}{5!\,2!}$

Step 1: First term

$$\frac{1440}{6\times 24} = \frac{1440}{144} = 10$$

Step 2: Second term

$$\frac{2400}{120\times 2} = \frac{2400}{240} = 10$$

Step 3: Sum

$$10 + 10 = 20$$

Answer: 20

(iv) $\frac{(n+2)!}{(n+1)!}$

Step 1: Use the property $(n+2)! = (n+2)(n+1)!$

Step 2: $\frac{(n+2)(n+1)!}{(n+1)!} = n+2$

Answer: $n+2$

Question 2: Write in factorial form

(i) $n^3 - n$

Step 1: Factor

$$n^3 - n = n(n^2-1) = n(n-1)(n+1)$$

Step 2: Express in factorial form

$$n(n-1)(n+1) = (n+1)n(n-1)$$

Answer: $(n+1) \cdot n \cdot (n-1)$

(ii) $\mathbf{n(n-1)(n-2)\cdots(n-r+1)}$

Step 1: This is the product of r consecutive terms starting from n

Step 2: $n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$

Answer: $\frac{n!}{(n-r)!}$

Question 3: Find n, if $(\mathbf{n+4})! = \mathbf{3024 \cdot n!}$

Step 1: Expand LHS

$$(n+4)(n+3)(n+2)(n+1)n! = 3024 \cdot n!$$

Step 2: Cancel $n!$ from both sides (assuming $n! \neq 0$)

$$(n+4)(n+3)(n+2)(n+1) = 3024$$

Step 3: Find factors of 3024

- $3024 = 7 \times 432 = 7 \times 6 \times 72 = 7 \times 6 \times 8 \times 9$
- $3024 = 9 \times 8 \times 7 \times 6$

Step 4: Compare

$$(n+4)(n+3)(n+2)(n+1) = 9 \times 8 \times 7 \times 6$$

- $n + 4 = 9 \rightarrow n = 5$
- Check: $5 + 3 = 8, 5 + 2 = 7, 5 + 1 = 6$

Answer: $n = 5$

Question 4: If $\frac{1}{7!} + \frac{1}{8!} = \frac{x}{9!}$, find x

Step 1: Express all terms with denominator 9!

$$\frac{1}{7!} = \frac{9 \times 8}{9!} = \frac{72}{9!} \frac{1}{8!} = \frac{9}{9!}$$

Step 2: Add

$$\frac{72}{9!} + \frac{9}{9!} = \frac{81}{9!}$$

Step 3: Therefore $x = 81$

Answer: $x = 81$

Question 5: Prove $\frac{(2n+1)!}{n!} = [1 \cdot 3 \cdot 5 \cdots (2n-1)(2n+1)]2^n$

Step 1: Expand LHS

$$\frac{(2n+1)!}{n!} = \frac{(2n+1)(2n)(2n-1)(2n-2)\cdots 3\cdot 2\cdot 1}{n!}$$

Step 2: Separate odd and even factors

- Odd factors: $(2n+1)(2n-1)(2n-3)\cdots 3\cdot 1$
- Even factors: $(2n)(2n-2)(2n-4)\cdots 2$

Step 3: Even factors product

$$(2n)(2n-2)(2n-4)\cdots 2 = 2^n \cdot n!$$

Step 4: Therefore

$$\frac{(2n+1)!}{n!} = \frac{[(2n+1)(2n-1)\cdots 1][2^n\cdot n!]}{n!} = [1\cdot 3\cdot 5\cdots(2n-1)(2n+1)]2^n$$

Therefore, the identity is proven

Question 6: Express as a single fraction

$$\frac{(n+2)!}{(r+2)!} + \frac{(n+1)!}{(r+1)!}$$

Step 1: Write with common denominator

$$= \frac{(n+2)!}{(r+2)!} + \frac{(r+2)(n+1)!}{(r+2)!}$$

Step 2: Factor

$$= \frac{(n+1)!}{(r+2)!}[(n+2)+(r+2)] = \frac{(n+1)!(n+r+4)}{(r+2)!}$$

Answer: $\frac{(n+1)!(n+r+4)}{(r+2)!}$

Question 7: Four distinct colored balls and four boxes of same colors

Step 1: Total ways to place 4 balls in 4 boxes (one each) $= 4! = 24$

Step 2: We need arrangements where no ball goes to its own colored box

- This is a derangement problem for 4 items

Step 3: Number of derangements of 4 items:

$$D_4 = 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 24\left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 24\left(0 + \frac{12}{24} - \frac{4}{24} + \frac{1}{24}\right) = 24\left(\frac{9}{24}\right) = 9$$

Answer: 9 ways

Summary Table

1(i)	$\frac{101}{40320}$
1(ii)	$\frac{121}{2177280}$
1(iii)	20
1(iv)	$n+2$
2(i)	$(n+1)n(n-1)$
2(ii)	$\frac{n!}{(n-r)!}$
3	$n=5$
4	$x=81$
5	Proven
6	$\frac{(n+1)!(n+r+4)}{(r+2)!}$
7	9 ways