EXERCISE 7.2 - PERMUTATIONS

Complete Solutions

Question 1: Evaluate the following

Formula: ${}^nP_r = \frac{n!}{(n-r)!}$

(i) $^{10}\mathbf{P_5}$

$$^{10}P_5 = \frac{10!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

Answer: 30240

(ii) $^{5}\mathbf{P_2}$

$$^5P_2 = 5 \times 4 = 20$$

Answer: 20

(iii) $^{7}\mathbf{P_3}$

$$^7P_3 = 7 \times 6 \times 5 = 210$$

Answer: 210

(iv) $^{10}\mathbf{P_3}$

$$^{10}P_3 = 10 \times 9 \times 8 = 720$$

Answer: 720

Question 2: Find the value of n

(i) $^{n}\mathbf{P_3} = \mathbf{504}$

$$n(n-1)(n-2) = 504$$

- $504 = 9 \times 8 \times 7$
- $n = 9$

Answer: $n = 9$

(ii) $^{n}\mathbf{P_2} = \mathbf{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}$

$$n(n-1) = 15 \times 14 \times 13 \times 12 \times 11 n(n-1) = 360360$$

- $600 \times 599 = 359400$
- $601 \times 600 = 360600$
- This doesn't match exactly. Let's recheck.

Actually, $15 \times 14 \times 13 \times 12 \times 11 = \frac{15!}{10!} = {}^{15}P_5$

So ${}^{n}P_2 = {}^{15}P_5$ implies $n(n-1) = 15 \times 14 \times 13 \times 12 \times 11$

But ${}^{15}P_5$ is not of the form ${}^{n}P_2$. The right side is a product of 5 terms, but ${}^{n}P_2$ has only 2 terms.

There seems to be an error in the problem statement.

(iii) ${}^{n}\mathbf{P}_2 = \mathbf{540 \cdot 1}$

$$n(n-1) = 540$$

- $24 \times 23 = 552$
- $23 \times 22 = 506$
- So $n = 24$ doesn't work exactly.

Check: $24 \times 23 = 552$, $23 \times 22 = 506$

No integer n satisfies $n(n-1) = 540$ exactly.

Question 3: Prove from first principle

(i) ${}^{n}\mathbf{P}_r = \mathbf{n} \cdot {}^{n-1}\mathbf{P}_{r-1}$

Step 1: LHS $= {}^{n}P_r = \frac{n!}{(n-r)!}$

Step 2: RHS $= n \cdot \frac{(n-1)!}{((n-1)-(r-1))!} = n \cdot \frac{(n-1)!}{(n-r)!}$

Step 3: Since $n! = n(n-1)!$, LHS $= \frac{n(n-1)!}{(n-r)!} = n \cdot \frac{(n-1)!}{(n-r)!} =$ RHS

Therefore, ${}^{n}P_r = n \cdot {}^{n-1}P_{r-1}$

(ii) ${}^{n}\mathbf{P}_r = {}^{n-1}\mathbf{P}_r + \mathbf{r} \cdot {}^{n-1}\mathbf{P}_{r-1}$

Step 1: RHS

$$ {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r \cdot \frac{(n-1)!}{(n-r)!}$$

Step 2: Common denominator $(n-r)!$

$$= \frac{(n-1)!(n-r)}{(n-r)!} + \frac{r(n-1)!}{(n-r)!} = \frac{(n-1)!(n-r+r)}{(n-r)!} = \frac{(n-1)! \cdot n}{(n-r)!} = \frac{n!}{(n-r)!} = {}^{n}P_r$$

Therefore, ${}^{n}P_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

Question 4: Words from letters using all letters

(i) PYTHON

- 6 distinct letters
- Number of words $= 6! = 720$

Answer: 720

(ii) NETWORK

- 7 distinct letters

- Number of words = $7! = 5040$

Answer: 5040

(iii) COMPUTER

- 8 distinct letters
- Number of words = $8! = 40320$

Answer: 40320

Question 5: Signals from 6 flags using 2 at a time

Step 1: Choose 2 flags from 6 and arrange them

- Number of signals = ${}^6P_2 = 6 \times 5 = 30$

Answer: 30

Question 6: Signals from 5 flags using any number

Step 1: Using 1 flag: ${}^5P_1 = 5$
Step 2: Using 2 flags: ${}^5P_2 = 5 \times 4 = 20$
Step 3: Using 3 flags: ${}^5P_3 = 5 \times 4 \times 3 = 60$
Step 4: Using 4 flags: ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$
Step 5: Using 5 flags: ${}^5P_5 = 5! = 120$

Step 6: Total $= 5 + 20 + 60 + 120 + 120 = 325$

Answer: 325

Question 7: 4-digit numbers with distinct odd digits

Step 1: Odd digits: 1, 3, 5, 7, 9 (5 digits)

- We need 4-digit numbers using distinct digits from these 5
- Number $= {}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$

Answer: 120

Question 8: Numbers between 100 and 1000 using digits 0,1,2,3,4,5 without repetition

Part 1: Total numbers

Step 1: 3-digit numbers (between 100 and 1000)

- Hundreds place: cannot be 0, so 5 choices (1,2,3,4,5)
- Tens place: remaining 5 digits (including 0)
- Units place: remaining 4 digits
- Total $= 5 \times 5 \times 4 = 100$

Answer: 100 numbers

Part 2: Numbers divisible by 5

Step 1: Units place must be 0 or 5

Case 1: Units = 0

- Hundreds: 5 choices (1,2,3,4,5)
- Tens: remaining 4 choices

- Total $= 5 \times 4 = 20$

Case 2: Units $= 5$

- Hundreds: cannot be 0, so 4 choices (1,2,3,4)
- Tens: remaining 4 choices (including 0)
- Total $= 4 \times 4 = 16$

Step 2: Total divisible by $5 = 20 + 16 = 36$

Answer: 36 numbers

Question 9: Numbers greater than 35000 from digits 1,2,3,4,5,6

Step 1: 5-digit numbers from 6 digits without repetition

Step 2: First digit choices:

Case 1: First digit $= 3$

- Second digit must be 5 (to make number > 35000)
- Second digit: 5 or 6 (2 choices)
- Remaining 3 positions: ${}^4P_3 = 4 \times 3 \times 2 = 24$
- Total $= 2 \times 24 = 48$

Case 2: First digit > 3 (4, 5, or 6) $\rightarrow$ 3 choices

- Remaining 4 positions: ${}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$
- Total $= 3 \times 120 = 360$

Step 3: Total $= 48 + 360 = 408$

Answer: 408

Question 10: 5-digit numbers from 1,2,4,6,8

(i) Digits 2 and 8 next to each other

Step 1: Treat (2,8) as one unit

- Number of units: 4 (the pair + 3 other digits)
- Arrange 4 units: $4! = 24$
- Within the pair, 2 and 8 can be arranged: 2 ways
- Total $= 24 \times 2 = 48$

Answer: 48

(ii) Digits 2 and 8 not next to each other

Step 1: Total arrangements without restriction $= 5! = 120$

Step 2: Arrangements with 2 and 8 together $= 48$

Step 3: Arrangements with 2 and 8 not together $= 120 - 48 = 72$

Answer: 72

Question 11: 6-digit numbers from 0,1,2,3,4,5 without repetition

Part 1: Total 6-digit numbers

Step 1: First digit cannot be 0 → 5 choices
Step 2: Remaining 5 positions: 5! = 120 ways
Step 3: Total = 5 × 120 = 600

Answer: 600

Part 2: Numbers with 0 at tens place

Step 1: 0 fixed at tens place

- First digit: cannot be 0, so 5 choices (1,2,3,4,5)
- Remaining 4 positions: 4! = 24 ways
- Total = 5 × 24 = 120

Answer: 120

Question 12: 5-digit multiples of 5 from 2,3,5,7,9

Step 1: For multiple of 5, last digit must be 5 (only digit divisible by 5)

- Last digit fixed as 5

Step 2: Remaining 4 positions from 4 digits (2,3,7,9)

- Number = 4! = 24

Answer: 24

Question 13: 8 different books including 2 English not together

Step 1: Total arrangements without restriction = 8!

Step 2: Arrangements with English books together:

- Treat 2 English books as one unit → 7 units
- Arrange 7 units: 7!
- Arrange English books within unit: 2!
- Total together = 7! × 2!

Step 3: Arrangements not together = 8! − 7! × 2! = 40320 − 10080 = 30240

Answer: 30240

Question 14: 3 English and 5 Urdu books, same subject together

Step 1: Treat English books as one unit, Urdu books as one unit

- 2 units to arrange: 2! = 2

Step 2: Arrange 3 English books within unit: 3! = 6
Step 3: Arrange 5 Urdu books within unit: 5! = 120

Step 4: Total = 2 × 6 × 120 = 1440

Answer: 1440

Question 15: 5 boys and 4 girls in alternate seats

Step 1: Total seats = 9
Step 2: To have alternate seating, pattern must be B G B G B G B G B

- Boys occupy odd positions (5 positions): 5! = 120

- Girls occupy even positions (4 positions): $4! = 24$
  Step 3: Total $= 120 \times 24 = 2880$

Answer: 2880

Summary Table

1(i)	30240
1(ii)	20
1(iii)	210
1(iv)	720
2(i)	$n = 9$
2(ii)	Invalid question
2(iii)	No integer solution
4(i)	720
4(ii)	5040
4(iii)	40320
5	30
6	325
7	120
8	100 (total), 36 (divisible by 5)
9	408
10(i)	48
10(ii)	72
11	600 (total), 120 (0 at tens)
12	24
13	30240
14	1440
15	2880