EXERCISE 7.4 - COMBINATIONS

Complete Solutions

Question 1: Evaluate the following

Formula: ${}^nC_r = \frac{n!}{r!(n-r)!}$

(i) $^{50}C_{30}$

$$^{50}C_{30} = ^{50}C_{20} = \frac{50!}{20! \cdot 30!}$$

Answer: $\frac{50!}{20! \cdot 30!}$ (or a very large number $\;4.712 \times 10^{13}$)

(ii) $^{1000}C_0$

$$^{1000}C_0 = 1$$

Answer: 1

(iii) $^{10}C_7$

$$^{10}C_7 = ^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

Answer: 120

(iv) $^{20}C_{17}$

$$^{20}C_{17} = ^{20}C_3 = \frac{20 \times 19 \times 18}{3 \times 2 \times 1} = 1140$$

Answer: 1140

Question 2

(i) If $^n\mathbf{C_2}$:$^5\mathbf{C_2}$= 15 : 1, find n

Step 1: Write the ratio

$$\frac{^nC_2}{^5C_2} = \frac{15}{1}$$

Step 2: ${}^nC_2 = \frac{n(n-1)}{2}$, ${}^5C_2 = 10$

$$\frac{\frac{n(n-1)}{2}}{10} = 15\frac{n(n-1)}{20} = 15n(n-1) = 300n^2 - n - 300 = 0$$

Step 3: Factor

$$(n-25)(n+24) = 0n = 25 \text{ or } n = -24 \text{ (reject)}$$

Answer: $n = 25$

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(ii) If $^{n}P_{r}= 120$ and $^{n}C_{r}= 20$, find r

Step 1: $^{n}P_{r} = r! \cdot^{n} C_{r}$

$$120 = r! \cdot 20r! = 6r = 3$$

Answer: $r = 3$

Question 3: Find values of n and r

(i) $^{n}C_{r}= 56,^{n}P_{r}= 336$

Step 1: $^{n}P_{r} = r! \cdot^{n} C_{r}$

$$336 = r! \cdot 56r! = 6r = 3$$

Step 2: $^{n}C_{3} = 56$

$$\frac{n(n-1)(n-2)}{6} = 56n(n-1)(n-2) = 3368 \times 7 \times 6 = 336n = 8$$

Answer: $n = 8, r = 3$

(ii) $^{n-1}C_{r-1}:^{n}C_{r}:^{n+1}C_{r+1}= 1:3:7$

Step 1: Let $^{n-1}C_{r-1} = k$, $^{n}C_{r} = 3k$, $^{n+1}C_{r+1} = 7k$

Step 2: $^{n}C_{r} = \frac{n}{r} \cdot^{n-1} C_{r-1}$

$$3k = \frac{n}{r} \cdot k\frac{n}{r} = 3n = 3r$$

Step 3: $^{n+1}C_{r+1} = \frac{n+1}{r+1} \cdot^{n} C_{r}$

$$7k = \frac{n+1}{r+1} \cdot 3k7 = \frac{3(n+1)}{r+1}7(r+1) = 3(3r+1)7r+7 = 9r+34 = 2rr = 2$$

Step 4: $n = 3r = 6$

Answer: $n = 6, r = 2$

Question 4: Prove the identities

(i) $^{n}C_{r}+^{n}C_{r-1}=^{n+1}C_{r}$

Step 1: LHS

$$\frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

Step 2: Common denominator $r!(n-r+1)!$

$$= \frac{n!(n-r+1)}{r!(n-r+1)!} + \frac{n! \cdot r}{r!(n-r+1)!} = \frac{n!(n-r+1+r)}{r!(n-r+1)!} = \frac{n!(n+1)}{r!(n+1-r)!} = \frac{(n+1)!}{r!(n+1-r)!} =^{n+1} C_{r}$$

Therefore, $^{n}C_{r} +^{n} C_{r-1} =^{n+1} C_{r}$

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(ii) $\mathbf{{}^nC_r : {}^nC_{r-1} = (n - r + 1) : r}$

Step 1: LHS

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{(r-1)!(n-r+1)!}{r!(n-r)!} = \frac{n-r+1}{r}$$

Therefore, ${}^nC_r :{}^n C_{r-1} = (n-r+1) : r$

Question 5: Product of r consecutive integers is divisible by r!

Step 1: Product of r consecutive integers:

$$n(n-1)(n-2)\cdots(n-r+1)$$

Step 2: This equals ${}^nP_r = r! \cdot{}^n C_r$

Step 3: Since ${}^nC_r$ is an integer, the product is divisible by $r!$

Therefore, product of r consecutive integers is divisible by $r!$

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Question 6: Five subjects selected out of eight

Step 1: Number of ways $= {}^8C_5 ={}^8 C_3 = \frac{8\times7\times6}{6} = 56$

Answer: 56

Question 7: Vowel arrangements from English alphabet

Step 1: Vowels: A, E, I, O, U (5 vowels)

Step 2: Number of possible arrangements of vowel letters $= 5! = 120$

Answer: 120

Question 8: 3 Desi from 6, 2 Chinese from 8

Step 1: Choose 3 Desi from 6: ${}^6C_3$
Step 2: Choose 2 Chinese from 8: ${}^8C_2$
Step 3: Total $= {}^6C_3 \times{}^8 C_2 = 20 \times 28 = 560$

Answer: 560

Question 9: 8 cards from 52, 3 black and 5 red

Step 1: Choose 3 black from 26: ${}^{26}C_3$
Step 2: Choose 5 red from 26: ${}^{26}C_5$
Step 3: Total $= {}^{26}C_3 \times{}^{26} C_5 = 2600 \times 65780 = 171028000$

Answer: 171,028,000

Question 10: Selection of 5 balls from 8 red and 7 green

(i) 3 red and 2 green

$${}^8C_3 \times{}^7 C_2 = 56 \times 21 = 1176$$

Answer: 1176

(ii) 1 red and 4 green

$$^{8}C_1 \times ^{7}C_4 = 8 \times 35 = 280$$

Answer: 280

(iii) 4 red and 1 green

$$^{8}C_4 \times ^{7}C_1 = 70 \times 7 = 490$$

Answer: 490

(iv) all the red balls (5 red balls selected)

$$^{8}C_5 = 56$$

Answer: 56

Question 11: Diagonals and triangles from 15-sided polygon

Diagonals

- Formula: $\frac{n(n-3)}{2}$
- For $n = 15$: $\frac{15\times12}{2} = 90$

Answer: 90 diagonals

Triangles

- Choose 3 vertices from 15: $^{15}C_3 = 455$

Answer: 455 triangles

Question 12: Number of sides if diagonals = 104

Step 1: $\frac{n(n-3)}{2} = 104$

$$n(n-3) = 208n^2 - 3n - 208 = 0(n-16)(n+13) = 0 \;\; n = 16 \text{ (reject } n = -13)$$

Answer: 16 sides

Question 13: Triangles from 15 points, 6 collinear

Step 1: Total triangles without restriction $= {}^{15}C_3 = 455$

Step 2: Triangles that cannot be formed (all 3 points on same line) $= {}^{6}C_3 = 20$

Step 3: Valid triangles $= 455 - 20 = 435$

Answer: 435

Question 14: Committee of 6 boys and 3 girls from 10 boys and 8 girls

Step 1: Choose 6 boys from 10: $^{10}C_6 = 210$
Step 2: Choose 3 girls from 8: $^{8}C_3 = 56$
Step 3: Total $= 210 \times 56 = 11760$

Answer: 11760

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Question 15: Committees of 7 from 10, including 2 particular persons

Step 1: 2 particular persons are already included
Step 2: Need 5 more from remaining 8 persons
Step 3: ${}^{8}C_5 = {}^{8}C_3 = 56$

Answer: 56

Question 16: Cricket team of 11 from 17 players

Total ways

$${}^{17}C_{11} = {}^{17}C_6 = 12376$$

Answer: 12376

Including a particular player

Step 1: That player is already included
Step 2: Need 10 more from remaining 16 players
Step 3: ${}^{16}C_{10} = {}^{16}C_6 = 8008$

Answer: 8008

Summary Table

1(i)	$\frac{50!}{20!30!}$
1(ii)	1
1(iii)	120
1(iv)	1140
2(i)	$n = 25$
2(ii)	$r = 3$
3(i)	$n = 8, r = 3$
3(ii)	$n = 6, r = 2$
4	Both proven
5	Proven
6	56
7	120
8	560
9	171,028,000
10(i)	1176
10(ii)	280
10(iii)	490
10(iv)	56
11	90 diagonals, 455 triangles
12	16 sides
13	435
14	11760
15	56
16	12376 (total), 8008 (with particular player)

EXERCISE 7.4 - COMBINATIONS (Continued)

Question 17: Committees of seven from 6 men and 8 women

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(i) with 3 women

Step 1: Choose 3 women from 8: ${}^8C_3 = 56$
Step 2: Choose 4 men from 6: ${}^6C_4 = 15$
Step 3: Total $= 56 \times 15 = 840$

Answer: 840

(ii) with at most 3 women

Step 1: At most 3 women means 0, 1, 2, or 3 women

Case 1: 0 women, 7 men: ${}^8C_0 \times {}^6C_7 = 1 \times 0 = 0$ (not possible as only 6 men)
Case 2: 1 woman, 6 men: ${}^8C_1 \times {}^6C_6 = 8 \times 1 = 8$
Case 3: 2 women, 5 men: ${}^8C_2 \times {}^6C_5 = 28 \times 6 = 168$
Case 4: 3 women, 4 men: ${}^8C_3 \times {}^6C_4 = 56 \times 15 = 840$

Step 2: Total $= 8 + 168 + 840 = 1016$

Answer: 1016

(iii) with at least 5 women

Step 1: At least 5 women means 5, 6, or 7 women (max 7 since committee has 7 members)

Case 1: 5 women, 2 men: ${}^8C_5 \times {}^6C_2 = 56 \times 15 = 840$
Case 2: 6 women, 1 man: ${}^8C_6 \times {}^6C_1 = 28 \times 6 = 168$
Case 3: 7 women, 0 men: ${}^8C_7 \times {}^6C_0 = 8 \times 1 = 8$

Step 2: Total $= 840 + 168 + 8 = 1016$

Answer: 1016

Question 18: Three sections, each with 3 questions, choose 5 questions, at least one from each section

Step 1: Total ways to choose 5 questions from 9 without restriction $= {}^9C_5 = 126$

Step 2: Subtract cases where at least one section is not represented

Case A: Only sections 1 and 2 (no section 3)

- Choose 5 from 6 questions: ${}^6C_5 = 6$

Case B: Only sections 1 and 3 (no section 2)

- Choose 5 from 6 questions: ${}^6C_5 = 6$

Case C: Only sections 2 and 3 (no section 1)

- Choose 5 from 6 questions: ${}^6C_5 = 6$

Case D: Only one section (choose 5 questions from 3 questions of one section)

- Not possible since each section has only 3 questions

Step 3: Using inclusion-exclusion:

$$\text{Valid} = 126 - (6 + 6 + 6) = 126 - 18 = 108$$

Answer: 108

Question 19: 8-character password with exactly 5 lowercase letters (a~f) and 3 digits (0~5)

(a) With repetition allowed

Step 1: Choose positions for 5 letters: ${}^8C_5 = 56$

Step 2: For each letter position: 6 choices (a,b,c,d,e,f) $\to 6^5$
Step 3: For each digit position: 6 choices (0,1,2,3,4,5) $\to 6^3$

Step 4: Total $= {}^8C_5 \times 6^5 \times 6^3 = 56 \times 6^8 = 56 \times 1679616 = 94058496$

Answer: 94,058,496

(b) Without repetition

Step 1: Choose positions for 5 letters: ${}^8C_5 = 56$

Step 2: Arrange 5 distinct letters from 6: ${}^6P_5 = 6! = 720$
Step 3: Arrange 3 distinct digits from 6: ${}^6P_3 = 6 \times 5 \times 4 = 120$

Step 4: Total $= {}^8C_5 \times {}^6P_5 \times {}^6P_3 = 56 \times 720 \times 120$

$$= 56 \times 86400 = 4838400$$

Answer: 4,838,400

Question 20: Selecting songs from two lists

Given:

- Teacher I: 10 distinct songs
- Teacher II: 10 different songs (no overlap)
- Need: 3 songs from each list

(i) Order/sequence is important

Step 1: Select and arrange 3 songs from Teacher I: ${}^{10}P_3 = 10 \times 9 \times 8 = 720$

Step 2: Select and arrange 3 songs from Teacher II: ${}^{10}P_3 = 720$

Step 3: Total $= 720 \times 720 = 518400$

Answer: 518,400

(ii) Order/sequence is not important

Step 1: Select 3 songs from Teacher I: ${}^{10}C_3 = 120$

Step 2: Select 3 songs from Teacher II: ${}^{10}C_3 = 120$

Step 3: Total $= 120 \times 120 = 14400$

Answer: 14,400

Summary Table

17(i)	840
17(ii)	1016
17(iii)	1016
18	108
19(a)	94,058,496
19(b)	4,838,400
20(i)	518,400
20(ii)	14,400

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