EXERCISE 9.1 - DIVISION OF POLYNOMIALS

Complete Solutions

Question 1: Find remainder and quotient

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Complete Solutions

Question 1: Find remainder and quotient

(i) $(\mathbf{3x^2 - x + 2}) \div (\mathbf{x - 1})$

Step 1: Use synthetic division with $x = 1$

$$\begin{array}{cccc} 1 & 3 & -1 & 2 \\ & & 3 & 2 \\ & 3 & 2 & 4 \end{array}$$

Step 2: Quotient $= 3x + 2$, Remainder $= 4$

Answer: Quotient: $3x + 2$, Remainder: 4

(ii) $(\mathbf{x^3 + 12x^2 - 3x + 4}) \div (\mathbf{x - 2})$

Step 1: Synthetic division with $x = 2$

$$\begin{array}{ccccc} 2 & 1 & 12 & -3 & 4 \\ & & 2 & 28 & 50 \\ & 1 & 14 & 25 & 54 \end{array}$$

Step 2: Quotient $= x^2 + 14x + 25$, Remainder $= 54$

Answer: Quotient: $x^2 + 14x + 25$, Remainder: 54

(iii) $(\mathbf{x^4 - 5x^3 - 8x^2 + 13x + 12}) \div (\mathbf{x - 6})$

Step 1: Synthetic division with $x = 6$

$$\begin{array}{cccccc} 6 & 1 & -5 & -8 & 13 & 12 \\ & & 6 & 6 & -12 & 6 \\ & 1 & 1 & -2 & 1 & 18 \end{array}$$

Step 2: Quotient $= x^3 + x^2 - 2x + 1$, Remainder $= 18$

Answer: Quotient: $x^3 + x^2 - 2x + 1$, Remainder: 18

(iv) $(\mathbf{5x^4 - 3x^3 + 2x^2 - 1}) \div (\mathbf{x^2 + 4})$

Step 1: Long division:

$$
\begin{array}{rrrrrr}
 & 5x^2 & -3x & -18 & & \\
x^2+4 & 5x^4 & -3x^3 & 2x^2 & 0x & -1 \\
 & 5x^4 & & 20x^2 & & \\
 & & -3x^3 & -18x^2 & 0x & \\
 & & -3x^3 & & -12x & \\
 & & & -18x^2 & 12x & -1 \\
 & & & -18x^2 & & -72 \\
 & & & & 12x & 71
\end{array}
$$

Step 3: Quotient $= 5x^2 - 3x - 18$, Remainder $= 12x + 71$

Answer: Quotient: $5x^2 - 3x - 18$, Remainder: $12x + 71$

(v) $(\mathbf{3x^4 - 5x^3 + 4x - 6}) \div (\mathbf{x^2 - 3x + 5})$

Step 1: Long division:

$$
\begin{array}{rrrrrr}
 & 3x^2 & +4x & -3 & & \\
x^2-3x+5 & 3x^4 & -5x^3 & 0x^2 & 4x & -6 \\
 & 3x^4 & -9x^3 & 15x^2 & & \\
 & & 4x^3 & -15x^2 & 4x & \\
 & & 4x^3 & -12x^2 & 20x & \\
 & & & -3x^2 & -16x & -6 \\
 & & & -3x^2 & 9x & -15 \\
 & & & & -25x & 9
\end{array}
$$

Step 3: Quotient $= 3x^2 + 4x - 3$, Remainder $= -25x + 9$

Answer: Quotient: $3x^2 + 4x - 3$, Remainder: $-25x + 9$

Question 2: Use remainder theorem

Remainder Theorem: When $P(x)$ is divided by $(x - a)$, remainder $= P(a)$

(i) $\mathbf{P(x) = x^2 + 5x + 6}$, divisor $= \mathbf{x - 2}$

- Remainder $= P(2) = 4 + 10 + 6 = 20$

Answer: 20

(ii) $\mathbf{P(x) = x^3 + 5x^2 + 6}$, divisor $= \mathbf{x + 1}$

- Remainder $= P(-1) = -1 + 5 + 6 = 10$

Answer: 10

(iii) $\mathbf{P(x) = x^4 + x^3 + x^2 + x + 1}$, divisor $= \mathbf{x - 1}$

- Remainder $= P(1) = 1 + 1 + 1 + 1 + 1 = 5$

Answer: 5

(iv) $\mathbf{P(x) = x^4 + x^3 + 1}$, divisor $= \mathbf{x + 3}$

- Remainder $= P(-3) = 81 - 27 + 1 = 55$

Answer: 55

(v) $\mathbf{P(x) = x^4 + x^3 + 2}$, divisor $= \mathbf{x + 2}$

- Remainder $= P(-2) = 16 - 8 + 2 = 10$

Answer: 10

Question 3: Use factor theorem

Factor Theorem: $(x - a)$ is a factor of $P(x)$ iff $P(a) = 0$

(i) $\mathbf{x + 1}$ and $\mathbf{x^2 - 1}$

- $P(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Answer: Yes, $x + 1$ is a factor

(ii) $\mathbf{x - 2}$ and $\mathbf{x^2 - 5x + 6}$

- $P(2) = 4 - 10 + 6 = 0$

Answer: Yes, $x - 2$ is a factor

(iii) $\mathbf{x + 1}$ and $\mathbf{x^3 + x^2 + x - 3}$

- $P(-1) = -1 + 1 - 1 - 3 = -4 \neq 0$

Answer: No, $x + 1$ is not a factor

(iv) $\mathbf{x - 2}$ and $\mathbf{x^3 + x^2 - 7x + 2}$

- $P(2) = 8 + 4 - 14 + 2 = 0$

Answer: Yes, $x - 2$ is a factor

(v) $\mathbf{x - 3}$ and $\mathbf{x^4 - 3x^3 + x^2 - x + 1}$

Note: Problem has $-3x^2 + x^2$ which simplifies to $-2x^2$

- $P(3) = 81 - 3(27) + 9 - 3 + 1 = 81 - 81 + 9 - 3 + 1 = 7 \neq 0$

Answer: No, $x - 3$ is not a factor

Question 4: Use synthetic division to show zero and factorize

(i) $\mathbf{x^2 - 7x + 6,\ x = 2}$

Step 1: Synthetic division with $x = 2$

$$\begin{array}{cccc} 2 & 1 & -7 & 6 \\ & & 2 & -10 \\ & 1 & -5 & -4 \end{array}$$

Remainder = -4 0, so $x = 2$ is NOT a zero.

Step 2: Factorize normally: $x^2 - 7x + 6 = (x - 1)(x - 6)$

Answer: Zeros are $x = 1$ and $x = 6$

(ii) $\mathbf{x^3 - 28x - 48,\ x = -4}$

Step 1: Synthetic division with $x = -4$

$$\begin{array}{c|cccc} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

Remainder = 0, so $x = -4$ is a zero.

Step 2: Quotient $= x^2 - 4x - 12$
Step 3: Factorize: $x^2 - 4x - 12 = (x-6)(x+2)$

Answer: $(x+4)(x-6)(x+2)$

(iii) $\mathbf{2x^4+7x^3-4x^2-27x-18,\ x=2, x=-3}$

Step 1: Test $x = 2$:

$$\begin{array}{c|ccccc} 2 & 2 & 7 & -4 & -27 & -18 \\ & & 4 & 22 & 36 & 18 \\ \hline & 2 & 11 & 18 & 9 & 0 \end{array}$$

Quotient: $2x^3 + 11x^2 + 18x + 9$

Step 2: Test $x = -3$ on quotient:

$$\begin{array}{c|cccc} -3 & 2 & 11 & 18 & 9 \\ & & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

Quotient: $2x^2 + 5x + 3$

Step 3: Factorize: $2x^2 + 5x + 3 = (2x+3)(x+1)$

Answer: $(x-2)(x+3)(2x+3)(x+1)$

Question 5: Synthetic division with $\mathbf{x = -3}$

$$P(x) = x^4 - 10x^2 - 2x + 4 \qquad \begin{array}{c|ccccc} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array}$$

Answer: Quotient: $x^3 - 3x^2 - x + 1$, Remainder: 1

Question 6: $\mathbf{x+1}$ and $\mathbf{x-2}$ are factors of $\mathbf{x^3-px^2+qx+2}$

Step 1: Since $x+1$ is a factor, $P(-1) = 0$:

$$-1 - p - q + 2 = 0$$

$1 - p - q = 0 \to p + q = 1$ ... (1)

Step 2: Since $x-2$ is a factor, $P(2) = 0$:

$$8 - 4p + 2q + 2 = 0$$

$10 - 4p + 2q = 0 \to -4p + 2q = -10 \to 2p - q = 5$ ... (2)

Step 3: Solve (1) and (2):
- From (1): $q = 1 - p$

- Substitute in (2): $2p-(1-p)=5 \to 3p-1=5 \to 3p=6 \to p=2$
- $q=1-2=-1$

Answer: $p=2, q=-1$

Question 7: Remainder is 16 when divided by $\mathbf{x+1}$

$$P(x)=4x^4+2x^3+kx^2+13$$

Step 1: $P(-1)=16$:

$$4(-1)^4+2(-1)^3+k(-1)^2+13=164-2+k+13=16k+15=16k=1$$

Answer: $k=1$

Question 8: Remainder is 7 when divided by $\mathbf{x+1}$

$$P(x)=x^3+x^2+x+k$$

Step 1: $P(-1)=7$:

$$-1+1-1+k=7k-1=7k=8$$

Answer: $k=8$

Summary Table

1(i)	$3x+2$, R=4
1(ii)	$x^2+14x+25$, R=54
1(iii)	x^3+x^2-2x+1, R=18
1(iv)	$5x^2-3x-18$, R=$12x+71$
1(v)	$3x^2+4x-3$, R=$-25x+9$
2(i)	20
2(ii)	10
2(iii)	5
2(iv)	55
2(v)	10
3(i)	Yes
3(ii)	Yes
3(iii)	No
3(iv)	Yes
3(v)	No
4(i)	$(x-1)(x-6)$
4(ii)	$(x+4)(x-6)(x+2)$
4(iii)	$(x-2)(x+3)(2x+3)(x+1)$
5	x^3-3x^2-x+1, R=1
6	$p=2, q=-1$
7	$k=1$
8	$k=8$

(i) $(\mathbf{3x^2-x+2})\div(\mathbf{x-1})$

Step 1: Use synthetic division with $x = 1$

1	3	-1	2
		3	2
	3	2	4

Step 2: Quotient $= 3x + 2$, Remainder $= 4$

Answer: Quotient: $3x + 2$, Remainder: 4

(ii) $(\mathbf{x^3+12x^2-3x+4}) \div (\mathbf{x-2})$

Step 1: Synthetic division with $x = 2$

2	1	12	-3	4
		2	28	50
	1	14	25	54

Step 2: Quotient $= x^2 + 14x + 25$, Remainder $= 54$

Answer: Quotient: $x^2 + 14x + 25$, Remainder: 54

(iii) $(\mathbf{x^4-5x^3-8x^2+13x+12}) \div (\mathbf{x-6})$

Step 1: Synthetic division with $x = 6$

6	1	-5	-8	13	12
		6	6	-12	6
	1	1	-2	1	18

Step 2: Quotient $= x^3 + x^2 - 2x + 1$, Remainder $= 18$

Answer: Quotient: $x^3 + x^2 - 2x + 1$, Remainder: 18

(iv) $(\mathbf{5x^4-3x^3+2x^2-1}) \div (\mathbf{x^2+4})$

Step 1: Long division:

	$5x^2$	$-3x$	-18		
x^2+4	$5x^4$	$-3x^3$	$2x^2$	$0x$	-1
	$5x^4$		$20x^2$		
		$-3x^3$	$-18x^2$	$0x$	
		$-3x^3$		$-12x$	
			$-18x^2$	$12x$	-1
			$-18x^2$		-72
				$12x$	71

Step 3: Quotient $= 5x^2 - 3x - 18$, Remainder $= 12x + 71$

Answer: Quotient: $5x^2 - 3x - 18$, Remainder: $12x + 71$

(v) $(\mathbf{3x^4-5x^3+4x-6}) \div (\mathbf{x^2-3x+5})$

Step 1: Long division:

	$3x^2$	$+4x$	-3		
x^2-3x+5	$3x^4$	$-5x^3$	$0x^2$	$4x$	-6
	$3x^4$	$-9x^3$	$15x^2$		
		$4x^3$	$-15x^2$	$4x$	
		$4x^3$	$-12x^2$	$20x$	
			$-3x^2$	$-16x$	-6
			$-3x^2$	$9x$	-15
				$-25x$	9

Step 3: Quotient $= 3x^2+4x-3$, Remainder $= -25x+9$

Answer: Quotient: $3x^2+4x-3$, Remainder: $-25x+9$

Question 2: Use remainder theorem

Remainder Theorem: When $P(x)$ is divided by $(x-a)$, remainder $= P(a)$

(i) $\mathbf{P(x) = x^2+5x+6}$, divisor $= \mathbf{x-2}$

- Remainder $= P(2) = 4+10+6 = 20$

Answer: 20

(ii) $\mathbf{P(x) = x^3+5x^2+6}$, divisor $= \mathbf{x+1}$

- Remainder $= P(-1) = -1+5+6 = 10$

Answer: 10

(iii) $\mathbf{P(x) = x^4+x^3+x^2+x+1}$, divisor $= \mathbf{x-1}$

- Remainder $= P(1) = 1+1+1+1+1 = 5$

Answer: 5

(iv) $\mathbf{P(x) = x^4+x^3+1}$, divisor $= \mathbf{x+3}$

- Remainder $= P(-3) = 81-27+1 = 55$

Answer: 55

(v) $\mathbf{P(x) = x^4+x^3+2}$, divisor $= \mathbf{x+2}$

- Remainder $= P(-2) = 16-8+2 = 10$

Answer: 10

Question 3: Use factor theorem

Factor Theorem: $(x-a)$ is a factor of $P(x)$ iff $P(a) = 0$

(i) $\mathbf{x+1}$ and $\mathbf{x^2-1}$

- $P(-1) = (-1)^2-1 = 1-1 = 0$

Answer: Yes, $x+1$ is a factor

(ii) $\mathbf{x-2}$ and $\mathbf{x^2-5x+6}$

- $P(2) = 4-10+6 = 0$

Answer: Yes, $x - 2$ is a factor

(iii) $\mathbf{x + 1}$ and $\mathbf{x^3 + x^2 + x - 3}$

- $P(-1) = -1 + 1 - 1 - 3 = -4 \neq 0$

Answer: No, $x + 1$ is not a factor

(iv) $\mathbf{x - 2}$ and $\mathbf{x^3 + x^2 - 7x + 2}$

- $P(2) = 8 + 4 - 14 + 2 = 0$

Answer: Yes, $x - 2$ is a factor

(v) $\mathbf{x - 3}$ and $\mathbf{x^4 - 3x^3 + x^2 - x + 1}$

Note: Problem has $-3x^2 + x^2$ which simplifies to $-2x^2$

- $P(3) = 81 - 3(27) + 9 - 3 + 1 = 81 - 81 + 9 - 3 + 1 = 7 \neq 0$

Answer: No, $x - 3$ is not a factor

Question 4: Use synthetic division to show zero and factorize

(i) $\mathbf{x^2 - 7x + 6, \ x = 2}$

Step 1: Synthetic division with $x = 2$

2	1	-7	6
		2	-10
	1	-5	-4

Remainder = -4   0, so $x = 2$ is NOT a zero.

Step 2: Factorize normally: $x^2 - 7x + 6 = (x - 1)(x - 6)$

Answer: Zeros are $x = 1$ and $x = 6$

(ii) $\mathbf{x^3 - 28x - 48, \ x = -4}$

Step 1: Synthetic division with $x = -4$

-4	1	0	-28	-48
		-4	16	48
	1	-4	-12	0

Remainder = 0, so $x = -4$ is a zero.

Step 2: Quotient = $x^2 - 4x - 12$
Step 3: Factorize: $x^2 - 4x - 12 = (x - 6)(x + 2)$

Answer: $(x + 4)(x - 6)(x + 2)$

(iii) $\mathbf{2x^4 + 7x^3 - 4x^2 - 27x - 18, \ x = 2, x = -3}$

Step 1: Test $x = 2$:

2	2	7	-4	-27	-18
		4	22	36	18
	2	11	18	9	0

Quotient: $2x^3 + 11x^2 + 18x + 9$

Step 2: Test $x = -3$ on quotient:

-3	2	11	18	9
		-6	-15	-9
	2	5	3	0

Quotient: $2x^2 + 5x + 3$

Step 3: Factorize: $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

Answer: $(x - 2)(x + 3)(2x + 3)(x + 1)$

Question 5: Synthetic division with $\mathbf{x = -3}$

$P(x) = x^4 - 10x^2 - 2x +$

-3	1	0	-10	-2	4
		-3	9	3	-3
	1	-3	-1	1	1

Answer: Quotient: $x^3 - 3x^2 - x + 1$, Remainder: 1

Question 6: $\mathbf{x + 1}$ and $\mathbf{x - 2}$ are factors of $\mathbf{x^3 - px^2 + qx + 2}$

Step 1: Since $x + 1$ is a factor, $P(-1) = 0$:

$$-1 - p - q + 2 = 0$$

$1 - p - q = 0 \rightarrow p + q = 1$ ... (1)

Step 2: Since $x - 2$ is a factor, $P(2) = 0$:

$$8 - 4p + 2q + 2 = 0$$

$10 - 4p + 2q = 0 \rightarrow -4p + 2q = -10 \rightarrow 2p - q = 5$ ... (2)

Step 3: Solve (1) and (2):
- From (1): $q = 1 - p$
- Substitute in (2): $2p - (1 - p) = 5 \rightarrow 3p - 1 = 5 \rightarrow 3p = 6 \rightarrow p = 2$
- $q = 1 - 2 = -1$

Answer: $p = 2, q = -1$

Question 7: Remainder is 16 when divided by $\mathbf{x + 1}$

$$P(x) = 4x^4 + 2x^3 + kx^2 + 13$$

Step 1: $P(-1) = 16$:

$$4(-1)^4 + 2(-1)^3 + k(-1)^2 + 13 = 164 - 2 + k + 13 = 16k + 15 = 16k = 1$$

Answer: $k = 1$

Question 8: Remainder is 7 when divided by $\mathbf{x + 1}$

$$P(x) = x^3 + x^2 + x + k$$

Step 1: $P(-1) = 7$:

$$-1 + 1 - 1 + k = 7k - 1 = 7k = 8$$

Answer: $k = 8$

Summary Table

1(i)	$3x + 2$, R=4
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1(iv)	$5x^2 - 3x - 18$, R=$12x + 71$
1(v)	$3x^2 + 4x - 3$, R=$-25x + 9$
2(i)	20
2(ii)	10
2(iii)	5
2(iv)	55
2(v)	10
3(i)	Yes
3(ii)	Yes
3(iii)	No
3(iv)	Yes
3(v)	No
4(i)	$(x-1)(x-6)$
4(ii)	$(x+4)(x-6)(x+2)$
4(iii)	$(x-2)(x+3)(2x+3)(x+1)$
5	$x^3 - 3x^2 - x + 1$, R=1
6	$p = 2, q = -1$
7	$k = 1$
8	$k = 8$

EXERCISE 9.1 - DIVISION OF POLYNOMIALS (Continued)

Question 9: Find p and q if $x + 1$ and $x - 2$ are factors of $x^3 + px^2 + qx + 3$

Step 1: Let $P(x) = x^3 + px^2 + qx + 3$

Step 2: Since $x + 1$ is a factor, by factor theorem:

$$P(-1) = 0(-1)^3 + p(-1)^2 + q(-1) + 3 = 0 - 1 + p - q + 3 = 0p - q + 2 = 0$$

$p - q = -2$ ... (1)

Step 3: Since $x - 2$ is a factor, by factor theorem:

$$P(2) = 0(2)^3 + p(2)^2 + q(2) + 3 = 08 + 4p + 2q + 3 = 04p + 2q + 11 = 0$$

$4p + 2q = -11$ ... (2)

Step 4: Solve the system of equations:
- From (1): $p = q - 2$
- Substitute in (2): $4(q-2) + 2q = -11$
- $4q - 8 + 2q = -11$
- $6q = -3$
- $q = -\frac{1}{2}$

Step 5: Find p:

$$p = q - 2 = -\frac{1}{2} - 2 = -\frac{5}{2}$$

Answer: $p = -\frac{5}{2}, q = -\frac{1}{2}$

Question 10: Find a and b if -2 and 2 are roots of $2x^3+4x^2+ax+b$

Step 1: Let $P(x) = 2x^3 + 4x^2 + ax + b$

Step 2: Since -2 is a root, by factor theorem:

$$P(-2) = 02(-2)^3 + 4(-2)^2 + a(-2) + b = 02(-8) + 4(4) - 2a + b = 0{-16} + 16 - 2a + b = 0{-2a} + b = 0$$