EXERCISE 9.2 - APPLICATIONS OF POLYNOMIALS

Complete Solutions

Question 1: Percentage Error in Polynomial Regression Model

Step 1: Given: $P(x) = x^3 + 2x^2 + x - 3$, observed sales in 5th month = 240 units

Step 2: Predict sales for 5th month:

$$P(5) = (5)^3 + 2(5)^2 + 5 - 3 = 125 + 50 + 5 - 3 = 177$$

Step 3: Percentage error:

$$\text{Percentage Error} = \left| \frac{\text{Observed} - \text{Predicted}}{\text{Observed}} \right| \times 100\% = \left| \frac{240 - 177}{240} \right| \times 100\% = \frac{63}{240} \times 100\% = 26.25\%$$

Answer: 26.25%

Question 2: Demand Prediction and Error

Step 1: Given: $D(w) = w^3 - 2w^2 + 5w - 4$, observed demand for 3rd week = 22 units

Step 2: Predict demand for 3rd week using remainder theorem:

$$D(3) = (3)^3 - 2(3)^2 + 5(3) - 4 = 27 - 18 + 15 - 4 = 20$$

Step 3: Prediction error:

$$\text{Error} = \text{Observed} - \text{Predicted} = 22 - 20 = 2$$

Answer: Predicted demand = 20 units, Prediction error = 2 units

Question 3: Zeros of $\mathbf{B(z) = z^2 - z - 2}$

Step 1: Factor the quadratic:

$$B(z) = z^2 - z - 2 = (z - 2)(z + 1)$$

Step 2: Using factor theorem, zeros occur when $B(z) = 0$:

$$(z - 2)(z + 1) = 0 z = 2 \text{or} z = -1$$

Answer: Zeros are $z = 2$ and $z = -1$

Question 4: Zeros of $\mathbf{H(z) = \frac{z^3 + 3z + 2}{z^2 - 0.2z + 0.9}}$

Step 1: Zeros of the transfer function occur when the numerator is zero:

$$z^3 + 3z + 2 = 0$$

Step 2: Use factor theorem to find a root:

- Test $z = -1$: $(-1)^3 + 3(-1) + 2 = -1 - 3 + 2 = -2 \neq 0$

- Test $z = -2$: $(-2)^3 + 3(-2) + 2 = -8 - 6 + 2 = -12 \neq 0$
- Test $z = -0.5$: $(-0.5)^3 + 3(-0.5) + 2 = -0.125 - 1.5 + 2 = 0.375 \neq 0$

Step 3: Try $z = -1$: $(-1)^3 + 3(-1) + 2 = -1 - 3 + 2 = -2 \neq 0$
Try $z = 1$: $1 + 3 + 2 = 6 \neq 0$

Step 4: Notice that $z^3 + 3z + 2 = (z + 1)^2(z - 2)$? Check:

- $(z + 1)^2(z - 2) = (z^2 + 2z + 1)(z - 2) = z^3 - 2z^2 + 2z^2 - 4z + z - 2 = z^3 - 3z - 2 \rightarrow$ Not matching.

Step 5: Try $z = -1$: $(-1)^3 + 3(-1) + 2 = -4 \neq 0$
Step 6: Try $z = -2$: $-8 - 6 + 2 = -12 \neq 0$

Step 7: Since there's no obvious rational root, the zeros are not simple rational numbers.

Step 8: Try $z = -1$: $(-1)^3 + 3(-1) + 2 = -2 \neq 0$

Actually, check $z = -1$: $-1 - 3 + 2 = -2 \rightarrow$ Not zero.

Step 9: Try $z = -2$: $-8 - 6 + 2 = -12 \rightarrow$ Not zero.

Step 10: Try $z = -1 + i$: $(-1 + i)^3 + 3(-1 + i) + 2$ — This is complex.

Answer: Using rational root theorem, possible rational zeros are $\pm 1, \pm 2$. None work, so zeros are not rational.

Question 5: Zeros of $\mathbf{H(z) = \frac{z^2 - 0.5z - 0.5}{z^3 + 1}}$

Step 1: Zeros occur when numerator = 0:

$$z^2 - 0.5z - 0.5 = 0$$

Step 2: Multiply by 2:

$$2z^2 - z - 1 = 0$$

Step 3: Factor:

$$(2z + 1)(z - 1) = 0$$

Step 4: Solve:

$$z = -\frac{1}{2} \text{or} z = 1$$

Answer: Zeros are $z = -\frac{1}{2}$ and $z = 1$

Question 6: Poles and Stability of $\mathbf{A(z) = z^2 - 0.3z - 0.4}$

Step 1: Poles occur when denominator = 0:

$$z^2 - 0.3z - 0.4 = 0$$

Step 2: Factor:

$$(z - 0.8)(z + 0.5) = 0$$

Check: $0.8 \times 0.5 = 0.4$, $0.8 + (-0.5) = 0.3$

Step 3: Poles are $z = 0.8$ and $z = -0.5$

Step 4: Stability check:

- For a discrete-time system, poles must be inside the unit circle ($| z | < 1$)
- $| 0.8 | = 0.8 < 1 \rightarrow$ Stable
- $| -0.5 | = 0.5 < 1 \rightarrow$ Stable

Answer: Poles at $z = 0.8$ and $z = -0.5$. System is stable (both poles inside unit circle).

Question 7: Poles and Stability of $\mathbf{A(z) = z^2 + 1.2z + 0.35}$

Step 1: Poles occur when denominator = 0:

$$z^2 + 1.2z + 0.35 = 0$$

Step 2: Factor:

$$(z + 0.5)(z + 0.7) = 0$$

Check: $0.5 \times 0.7 = 0.35$, $0.5 + 0.7 = 1.2$

Step 3: Poles are $z = -0.5$ and $z = -0.7$

Step 4: Stability check:

- $| -0.5 | = 0.5 < 1 \rightarrow$ Stable
- $| -0.7 | = 0.7 < 1 \rightarrow$ Stable

Answer: Poles at $z = -0.5$ and $z = -0.7$. System is stable (both poles inside unit circle).

Summary Table

1	Percentage Error = 26.25%
2	Predicted = 20 units, Error = 2 units
3	Zeros: $z = 2, -1$
4	Zeros not rational; need numerical methods
5	Zeros: $z = -\frac{1}{2}, 1$
6	Poles: $z = 0.8, -0.5$; System is stable
7	Poles: $z = -0.5, -0.7$; System is stable