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# Exercise 11.2

## Graphs of Trigonometric Functions

Complete Solutions -- Colorful Booklet Edition



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# Key Graphing Guidelines

## For functions of the form

- $y = A \sin(kx)$ : Amplitude =  $|A|$ , Period =  $\frac{2\pi}{|k|}$
- $y = A \cos(kx)$ : Amplitude =  $|A|$ , Period =  $\frac{2\pi}{|k|}$
- $y = \tan(kx)$ : Period =  $\frac{\pi}{|k|}$ , Vertical asymptotes at  $kx = \frac{\pi}{2} + n\pi$

## CHAPTER 1

### Question 1 -- Draw the Graph of Each Function

(i)  $y = -\sin 2x, \quad x \in [-2\pi, 2\pi]$

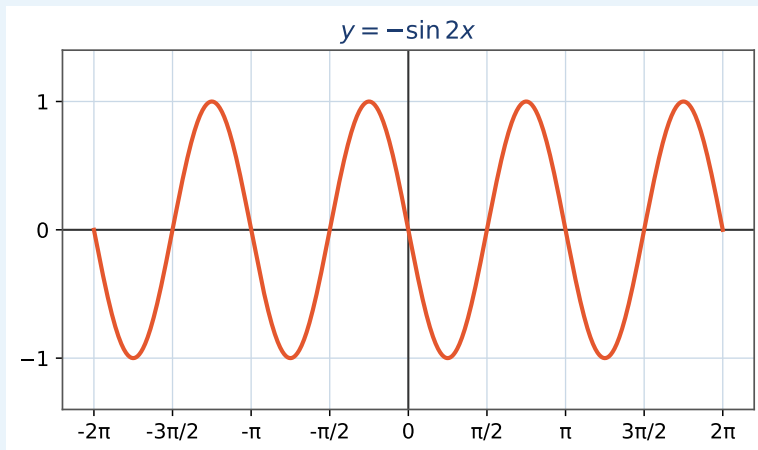
#### Step 1: Key Features

- Amplitude = 1
- Period =  $\frac{2\pi}{2} = \pi$
- Negative sign reflects the graph across the  $x$ -axis

#### Step 2: Key points for one period $[0, \pi]$

|            |   |         |         |          |        |
|------------|---|---------|---------|----------|--------|
| $x$        | 0 | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$  |
| $2x$       | 0 | $\pi/2$ | $\pi$   | $3\pi/2$ | $2\pi$ |
| $\sin 2x$  | 0 | 1       | 0       | -1       | 0      |
| $-\sin 2x$ | 0 | -1      | 0       | 1        | 0      |

**Step 3: Extend to  $[-2\pi, 2\pi]$**  – Since period =  $\pi$ , there are 4 complete periods in  $[-2\pi, 2\pi]$ .



**Graph Description:** A sine wave with period  $\pi$ , reflected across the  $x$ -axis, oscillating between  $-1$  and  $1$ .

(ii)  $y = 2 \cos 2x$ ,  $x \in [-2\pi, 2\pi]$

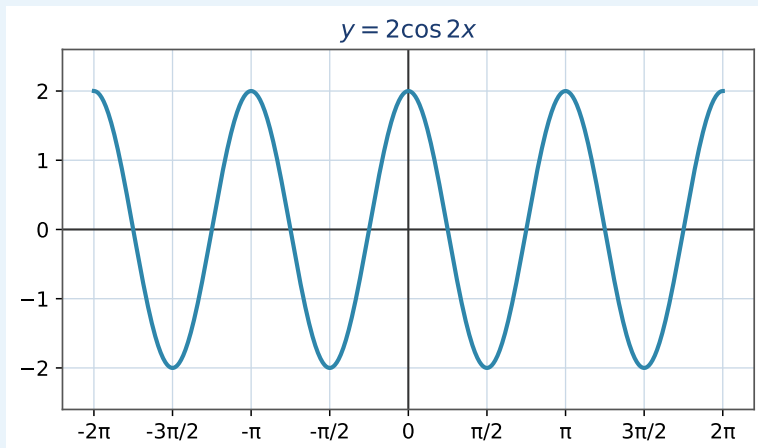
### Step 1: Key Features

- Amplitude = 2
- Period =  $\frac{2\pi}{2} = \pi$

### Step 2: Key points for one period $[0, \pi]$

|             |   |         |         |          |        |
|-------------|---|---------|---------|----------|--------|
| $x$         | 0 | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$  |
| $2x$        | 0 | $\pi/2$ | $\pi$   | $3\pi/2$ | $2\pi$ |
| $\cos 2x$   | 1 | 0       | -1      | 0        | 1      |
| $2 \cos 2x$ | 2 | 0       | -2      | 0        | 2      |

**Step 3: Extend to  $[-2\pi, 2\pi]$  – 4 complete periods in  $[-2\pi, 2\pi]$ .**



**Graph Description:** A cosine wave with period  $\pi$ , amplitude 2, oscillating between  $-2$  and  $2$ .

(iii)  $y = \tan 2x$ ,  $x \in [-\pi, \pi]$

### Step 1: Key Features

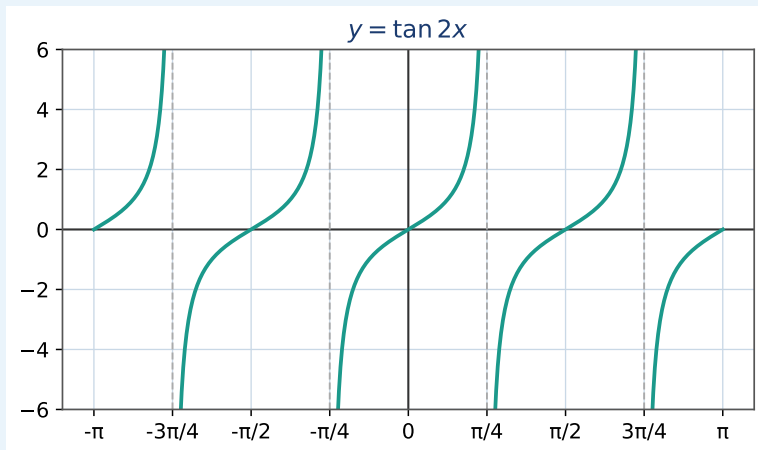
- Period =  $\frac{\pi}{2}$
- Vertical asymptotes where  $2x = \frac{\pi}{2} + n\pi \Rightarrow x = \frac{\pi}{4} + \frac{n\pi}{2}$

### Step 2: Vertical asymptotes in $[-\pi, \pi]$

$$x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

### Step 3: Key points between asymptotes – Between $-\pi/4$ and $\pi/4$ :

|           |          |     |         |
|-----------|----------|-----|---------|
| $x$       | $-\pi/8$ | $0$ | $\pi/8$ |
| $2x$      | $-\pi/4$ | $0$ | $\pi/4$ |
| $\tan 2x$ | $-1$     | $0$ | $1$     |



**Graph Description:** A tangent curve with period  $\pi/2$ , vertical asymptotes at  $x = \pi/4 + n\pi/2$ , passing through the origin with slope 2.

(iv)  $y = \tan(x/2)$ ,  $x \in [-2\pi, 2\pi]$

### Step 1: Key Features

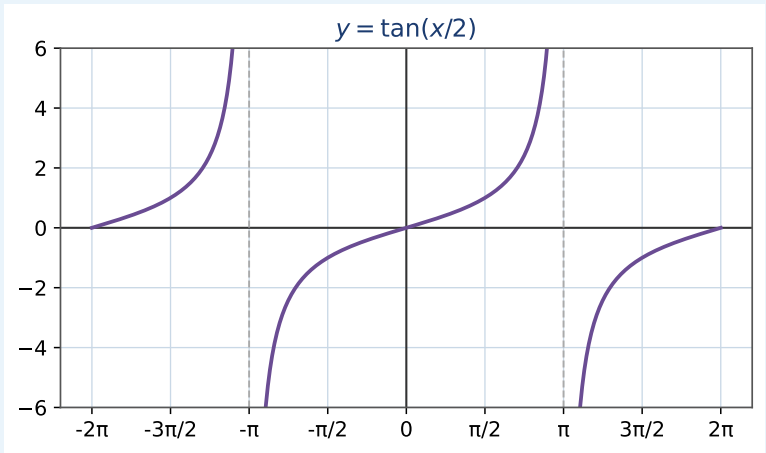
- Period =  $\frac{\pi}{1/2} = 2\pi$
- Vertical asymptotes where  $\frac{x}{2} = \frac{\pi}{2} + n\pi \Rightarrow x = \pi + 2n\pi$

### Step 2: Vertical asymptotes in $[-2\pi, 2\pi]$

$$x = -\pi, \pi$$

### Step 3: Key points

| $x$         | $-\pi$   | $-\pi/2$ | $0$ | $\pi/2$ | $\pi$   |
|-------------|----------|----------|-----|---------|---------|
| $x/2$       | $-\pi/2$ | $-\pi/4$ | $0$ | $\pi/4$ | $\pi/2$ |
| $\tan(x/2)$ | undef.   | $-1$     | $0$ | $1$     | undef.  |



**Graph Description:** A tangent curve with period  $2\pi$ , vertical asymptotes at  $x = \pm\pi$ .

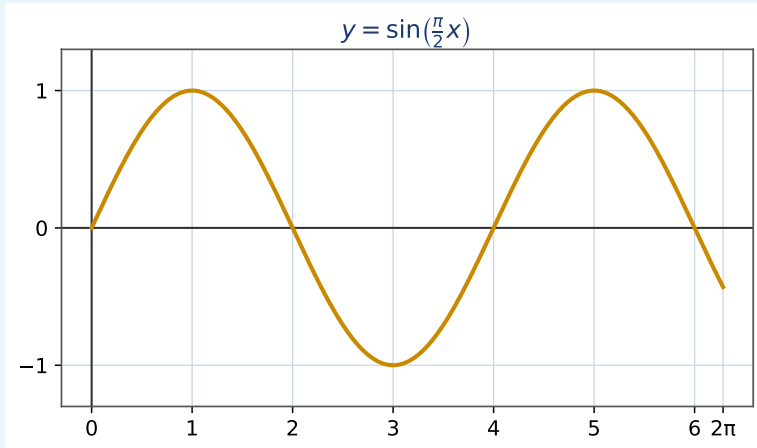
(v)  $y = \sin\left(\frac{\pi}{2}x\right), \quad x \in [0, 2\pi]$

Step 1: Key Features

- Period =  $\frac{2\pi}{\pi/2} = 4$
- The interval  $[0, 2\pi]$  is about 6.28 units wide

Step 2: Key points

|                  |   |         |       |          |        |          |        |             |
|------------------|---|---------|-------|----------|--------|----------|--------|-------------|
| $x$              | 0 | 1       | 2     | 3        | 4      | 5        | 6      | $2\pi$      |
| $\frac{\pi}{2}x$ | 0 | $\pi/2$ | $\pi$ | $3\pi/2$ | $2\pi$ | $5\pi/2$ | $3\pi$ | $\pi$       |
| $\sin(\cdot)$    | 0 | 1       | 0     | -1       | 0      | 1        | 0      | $\approx 0$ |



**Graph Description:** A sine wave with period 4, completing about 1.57 periods in the given interval.

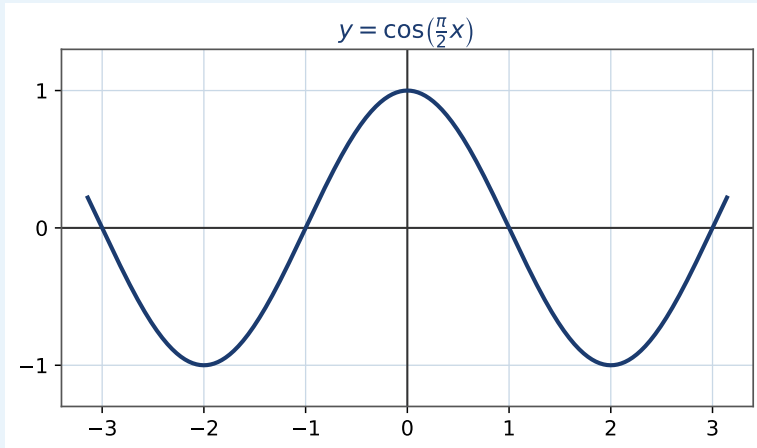
(vi)  $y = \cos\left(\frac{\pi}{2}x\right), \quad x \in [-\pi, \pi]$

### Step 1: Key Features

- Period =  $\frac{2\pi}{\pi/2} = 4$

### Step 2: Key points

|                  |           |        |          |   |         |       |          |
|------------------|-----------|--------|----------|---|---------|-------|----------|
| $x$              | -3        | -2     | -1       | 0 | 1       | 2     | 3        |
| $\frac{\pi}{2}x$ | $-3\pi/2$ | $-\pi$ | $-\pi/2$ | 0 | $\pi/2$ | $\pi$ | $3\pi/2$ |
| $\cos(\cdot)$    | 0         | -1     | 0        | 1 | 0       | -1    | 0        |



**Graph Description:** A cosine wave with period 4, showing part of two periods in  $[-\pi, \pi]$ .

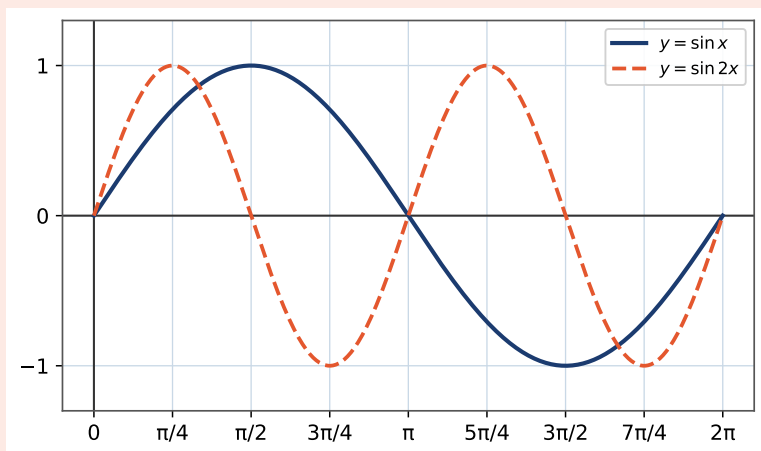
## CHAPTER 2

### Question 2 -- Graphs on the Same Axes

(i)  $y = \sin x$  and  $y = \sin 2x$

**Step 1:**  $y = \sin x$  – Period =  $2\pi$ , Amplitude = 1. Passes through:  $(0, 0)$ ,  $(\frac{\pi}{2}, 1)$ ,  $(\pi, 0)$ ,  $(\frac{3\pi}{2}, -1)$ ,  $(2\pi, 0)$ .

**Step 2:**  $y = \sin 2x$  – Period =  $\pi$ , Amplitude = 1. Passes through:  $(0, 0)$ ,  $(\frac{\pi}{4}, 1)$ ,  $(\frac{\pi}{2}, 0)$ ,  $(\frac{3\pi}{4}, -1)$ ,  $(\pi, 0)$ .

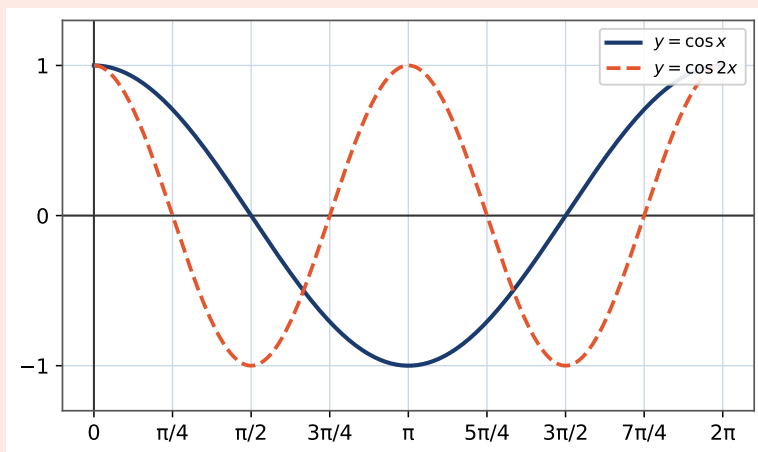


**Graph Description:** Both are sine waves with amplitude 1.  $\sin 2x$  oscillates twice as fast as  $\sin x$ , completing 2 full cycles in the same interval where  $\sin x$  completes 1 cycle.

(ii)  $y = \cos x$  and  $y = \cos 2x$

**Step 1:**  $y = \cos x$  – Period =  $2\pi$ , Amplitude = 1. Passes through:  $(0, 1)$ ,  $(\frac{\pi}{2}, 0)$ ,  $(\pi, -1)$ ,  $(\frac{3\pi}{2}, 0)$ ,  $(2\pi, 1)$ .

**Step 2:**  $y = \cos 2x$  – Period =  $\pi$ , Amplitude = 1. Passes through:  $(0, 1)$ ,  $(\frac{\pi}{4}, 0)$ ,  $(\frac{\pi}{2}, -1)$ ,  $(\frac{3\pi}{4}, 0)$ ,  $(\pi, 1)$ .



**Graph Description:** Both are cosine waves with amplitude 1.  $\cos 2x$  oscillates twice as fast as  $\cos x$ .

## CHAPTER 3

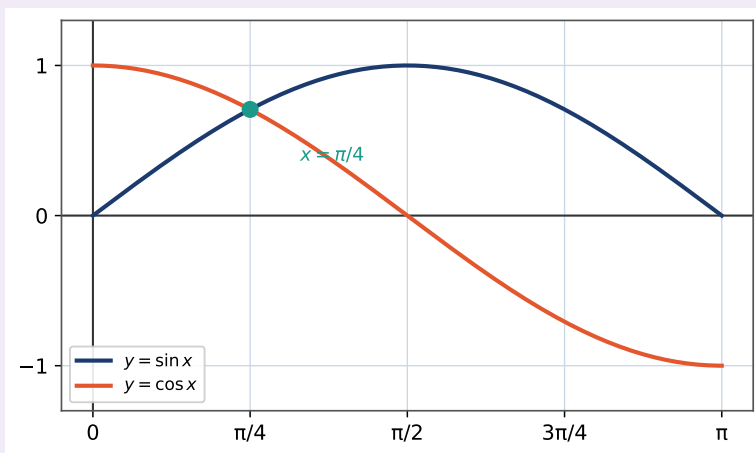
### Question 3 -- Solve Graphically

(i)  $\sin x = \cos x, \quad x \in [0, \pi]$

**Step 1:**  $\sin x = \cos x \Rightarrow \tan x = 1$  (when  $\cos x \neq 0$ ).

**Step 2:** In  $[0, \pi]$ ,  $\tan x = 1$  at  $x = \frac{\pi}{4}$ .

**Step 3:** Also check  $x = \pi/2 - \sin \frac{\pi}{2} = 1, \cos \frac{\pi}{2} = 0 \Rightarrow$  Not equal.



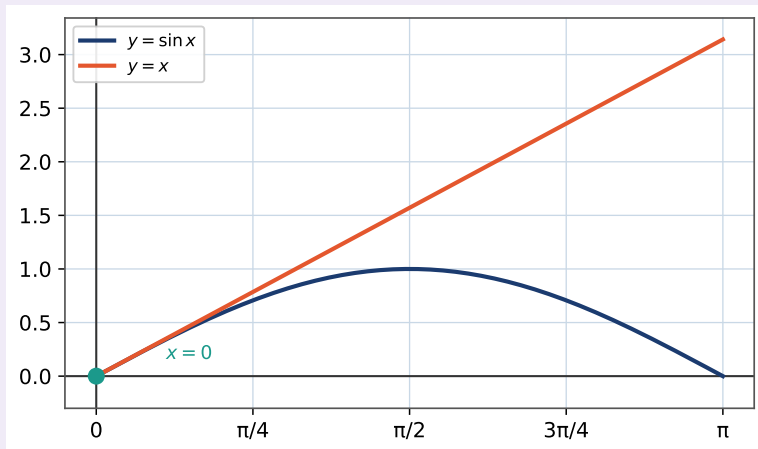
**Graphical Verification:** The graphs of  $y = \sin x$  and  $y = \cos x$  intersect at  $x = \pi/4$  in  $[0, \pi]$ .

**Answer:**  $x = \frac{\pi}{4}$

(ii)  $\sin x = x, \quad x \in [0, \pi]$

**Step 1:** The equation  $\sin x = x$  has only one solution in  $[0, \pi]$ :  $x = 0$ .

**Step 2: Verify** – At  $x = 0$ :  $\sin 0 = 0$ , RHS = 0 ✓. For  $x > 0$ ,  $\sin x < x$  (a well-known inequality for positive  $x$ ).



**Graphical Verification:** The line  $y = x$  and the curve  $y = \sin x$  intersect only at the origin in  $[0, \pi]$ .

**Answer:**  $x = 0$

## Summary of Results

### Question 1: Graph Features

| Function               | Period  | Amplitude | Asymptotes                           |
|------------------------|---------|-----------|--------------------------------------|
| $-\sin 2x$             | $\pi$   | 1         | None                                 |
| $2 \cos 2x$            | $\pi$   | 2         | None                                 |
| $\tan 2x$              | $\pi/2$ | —         | $x = \frac{\pi}{4} + \frac{n\pi}{2}$ |
| $\tan(x/2)$            | $2\pi$  | —         | $x = \pi + 2n\pi$                    |
| $\sin(\frac{\pi}{2}x)$ | 4       | 1         | None                                 |
| $\cos(\frac{\pi}{2}x)$ | 4       | 1         | None                                 |

### Question 3: Solutions

| Equation          | Solution in given interval |
|-------------------|----------------------------|
| $\sin x = \cos x$ | $x = \pi/4$                |
| $\sin x = x$      | $x = 0$                    |