Exercise 14.1

Vectors

Complete Solutions -- Colorful Booklet Edition

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Contents

Given Vectors

Question 1: Vector Operations

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}, \qquad \mathbf{v} = \mathbf{i} - 5\mathbf{j} - \mathbf{k}, \qquad \mathbf{w} = -4\mathbf{i} - \mathbf{j} + 7\mathbf{k}$$

CHAPTER 1

Vector Combinations (Question 1)

(i) $\mathbf{u} + 2\mathbf{v} + \mathbf{w}$

Step 1: $2\mathbf{v} = 2(\mathbf{i} - 5\mathbf{j} - \mathbf{k}) = 2\mathbf{i} - 10\mathbf{j} - 2\mathbf{k}$
Step 2: Add

$$\mathbf{u} + 2\mathbf{v} + \mathbf{w} = (3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + (2\mathbf{i} - 10\mathbf{j} - 2\mathbf{k}) + (-4\mathbf{i} - \mathbf{j} + 7\mathbf{k})$$

Step 3: Combine like terms

$$\begin{aligned} \mathbf{i}\text{ terms: } & 3 + 2 - 4 = 1 \\ \mathbf{j}\text{ terms: } & 2 - 10 - 1 = -9 \\ \mathbf{k}\text{ terms: } & -5 - 2 + 7 = 0 \end{aligned}$$

Answer: $\mathbf{i} - 9\mathbf{j}$

(ii) $\mathbf{v} - 3\mathbf{w}$

Step 1: $3\mathbf{w} = 3(-4\mathbf{i} - \mathbf{j} + 7\mathbf{k}) = -12\mathbf{i} - 3\mathbf{j} + 21\mathbf{k}$
Step 2:

$$\mathbf{v} - 3\mathbf{w} = (\mathbf{i} - 5\mathbf{j} - \mathbf{k}) - (-12\mathbf{i} - 3\mathbf{j} + 21\mathbf{k})$$

Step 3: Combine like terms

$$\begin{aligned} \mathbf{i}\text{ terms: } & 1 + 12 = 13 \\ \mathbf{j}\text{ terms: } & -5 + 3 = -2 \\ \mathbf{k}\text{ terms: } & -1 - 21 = -22 \end{aligned}$$

Answer: $13\mathbf{i} - 2\mathbf{j} - 22\mathbf{k}$

(iii) $3\mathbf{v} + \mathbf{w}$

Step 1: $3\mathbf{v} = 3(\mathbf{i} - 5\mathbf{j} - \mathbf{k}) = 3\mathbf{i} - 15\mathbf{j} - 3\mathbf{k}$

Step 2:

$$3\mathbf{v} + \mathbf{w} = (3\mathbf{i} - 15\mathbf{j} - 3\mathbf{k}) + (-4\mathbf{i} - \mathbf{j} + 7\mathbf{k})$$

Step 3: Combine like terms

$$\begin{aligned} \mathbf{i}\text{ terms: } & 3 - 4 = -1 \\ \mathbf{j}\text{ terms: } & -15 - 1 = -16 \\ \mathbf{k}\text{ terms: } & -3 + 7 = 4 \end{aligned}$$

Answer: $-\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

CHAPTER 2

Magnitude and Direction Cosines (Question 2)

(i) $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$

Step 1: Magnitude

$$|\mathbf{v}| = \sqrt{3^2 + (-2)^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$$

Step 2: Direction cosines

$$\cos\alpha = \frac{3}{7}, \qquad \cos\beta = -\frac{2}{7}, \qquad \cos\gamma = \frac{6}{7}$$

Answer: $|\mathbf{v}| = 7$, DC: $\left(\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}\right)$

(ii) $\mathbf{v} = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

Step 1: Magnitude

$$|\mathbf{v}| = \sqrt{(-4)^2 + 4^2 + 2^2} = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$$

Step 2: Direction cosines

$$\cos\alpha = -\frac{4}{6} = -\frac{2}{3}, \qquad \cos\beta = \frac{4}{6} = \frac{2}{3}, \qquad \cos\gamma = \frac{2}{6} = \frac{1}{3}$$

Answer: $|\mathbf{v}| = 6$, DC: $\left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(iii) $\mathbf{v} = -6\mathbf{i} + 8\mathbf{j}$

Step 1: Magnitude

$$|\mathbf{v}| = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Step 2: Direction cosines

$$\cos\alpha = -\frac{6}{10} = -\frac{3}{5}, \qquad \cos\beta = \frac{8}{10} = \frac{4}{5},$$
$$\cos\gamma = 0 \quad (\text{since the } \mathbf{k}\text{-component is } 0)$$

Answer: $|\mathbf{v}| = 10$, DC: $\left(-\frac{3}{5}, \frac{4}{5}, 0\right)$

CHAPTER 3

Solving for Unknowns (Questions 3--6)

Question 3: Find t

Given: $|2\mathbf{i} + (t-1)\mathbf{j} + t\mathbf{k}| = \sqrt{13}$

Step 1: Square both sides

$$\begin{aligned} 2^2 + (t-1)^2 + t^2 &= 13 \\ 4 + (t^2 - 2t + 1) + t^2 &= 13 \\ 2t^2 - 2t + 5 &= 13 \\ 2t^2 - 2t - 8 &= 0 \\ t^2 - t - 4 &= 0 \end{aligned}$$

Step 2: Solve using the quadratic formula

$$t = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

Answer: $t = \dfrac{1 \pm \sqrt{17}}{2}$

Question 4: Unit Vector in the Direction of $\mathbf{v} = -\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$

Step 1: Magnitude

$$|\mathbf{v}| = \sqrt{(-1)^2 + 4^2 + (-8)^2} = \sqrt{1 + 16 + 64} = \sqrt{81} = 9$$

Step 2: Unit vector

$$\hat{\mathbf{v}} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}}{9} = -\frac{1}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} - \frac{8}{9}\mathbf{k}$$

Answer: $-\frac{1}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} - \frac{8}{9}\mathbf{k}$

Question 5: Unit Vector Parallel to $4\mathbf{u} - 3\mathbf{v} + 2\mathbf{w}$

Given: $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{v} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{w} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

Step 1: $4\mathbf{u} = 8\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$

Step 2: $-3\mathbf{v} = -3(-\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) = 3\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}$

Step 3: $2\mathbf{w} = 6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$

Step 4: Add

$$\begin{aligned} 4\mathbf{u} - 3\mathbf{v} + 2\mathbf{w} &= (8\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) + (3\mathbf{i} - 12\mathbf{j} - 6\mathbf{k}) + (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) \\ &= (8 + 3 + 6)\mathbf{i} + (4 - 12 - 4)\mathbf{j} + (-12 - 6 + 2)\mathbf{k} \\ &= 17\mathbf{i} - 12\mathbf{j} - 16\mathbf{k} \end{aligned}$$

Step 5: Magnitude

$$\begin{aligned} |17\mathbf{i} - 12\mathbf{j} - 16\mathbf{k}| &= \sqrt{17^2 + (-12)^2 + (-16)^2} \\ &= \sqrt{289 + 144 + 256} = \sqrt{689} \end{aligned}$$

Step 6: Unit vector

$$\hat{\mathbf{v}} = \frac{17\mathbf{i} - 12\mathbf{j} - 16\mathbf{k}}{\sqrt{689}}$$

Answer: $\frac{17}{\sqrt{689}}\mathbf{i} - \frac{12}{\sqrt{689}}\mathbf{j} - \frac{16}{\sqrt{689}}\mathbf{k}$

Question 6: Vector with Given Magnitude and Direction

(i) Magnitude 5, parallel to $3\mathbf{i}+4\mathbf{j}-\mathbf{k}$

Step 1: $|3\mathbf{i}+4\mathbf{j}-\mathbf{k}| = \sqrt{9+16+1} = \sqrt{26}, \quad \text{so } \hat{\mathbf{v}} = \dfrac{3\mathbf{i}+4\mathbf{j}-\mathbf{k}}{\sqrt{26}}$

Step 2: $5\hat{\mathbf{v}} = \dfrac{15\mathbf{i}+20\mathbf{j}-5\mathbf{k}}{\sqrt{26}}$

Answer: $\dfrac{15}{\sqrt{26}}\mathbf{i} + \dfrac{20}{\sqrt{26}}\mathbf{j} - \dfrac{5}{\sqrt{26}}\mathbf{k}$

(ii) Magnitude 7, parallel to $-\mathbf{i}+\mathbf{j}+\mathbf{k}$

Step 1: $|-\mathbf{i}+\mathbf{j}+\mathbf{k}| = \sqrt{1+1+1} = \sqrt{3}, \quad \text{so } \hat{\mathbf{v}} = \dfrac{-\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$

Step 2: $7\hat{\mathbf{v}} = \dfrac{-7\mathbf{i}+7\mathbf{j}+7\mathbf{k}}{\sqrt{3}}$

Answer: $-\dfrac{7}{\sqrt{3}}\mathbf{i} + \dfrac{7}{\sqrt{3}}\mathbf{j} + \dfrac{7}{\sqrt{3}}\mathbf{k}$

Summary of Results -- Part I

Questions 1--6

Q.	Answer
1(i)	$\mathbf{i} - 9\mathbf{j}$
1(ii)	$13\mathbf{i} - 2\mathbf{j} - 22\mathbf{k}$
1(iii)	$-\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$
2(i)	$\lvert\mathbf{v}\rvert = 7,\ \left(\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}\right)$
2(ii)	$\lvert\mathbf{v}\rvert = 6,\ \left(-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$
2(iii)	$\lvert\mathbf{v}\rvert = 10,\ \left(-\frac{3}{5}, \frac{4}{5}, 0\right)$
3	$t = \dfrac{1 \pm \sqrt{17}}{2}$
4	$-\frac{1}{9}\mathbf{i} + \frac{4}{9}\mathbf{j} - \frac{8}{9}\mathbf{k}$
5	$\frac{17}{\sqrt{689}}\mathbf{i} - \frac{12}{\sqrt{689}}\mathbf{j} - \frac{16}{\sqrt{689}}\mathbf{k}$
6(i)	$\frac{15}{\sqrt{26}}\mathbf{i} + \frac{20}{\sqrt{26}}\mathbf{j} - \frac{5}{\sqrt{26}}\mathbf{k}$
6(ii)	$-\frac{7}{\sqrt{3}}\mathbf{i} + \frac{7}{\sqrt{3}}\mathbf{j} + \frac{7}{\sqrt{3}}\mathbf{k}$

CHAPTER 4

Triangles and Parallel Vectors (Questions 7--9)

Question 7: Sides of a Triangle

Given: $\mathbf{u} = x\mathbf{i} + 2\mathbf{j} + 3\mathbf{k},\ \mathbf{v} = \mathbf{i} + y\mathbf{j} - 3\mathbf{k},\ \mathbf{w} = -2\mathbf{i} - 3\mathbf{j}$

Step 1: For three vectors to represent the sides of a triangle (head to tail), their sum must be zero:

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$$

Step 2: Add the vectors

$$(x\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + (\mathbf{i} + y\mathbf{j} - 3\mathbf{k}) + (-2\mathbf{i} - 3\mathbf{j}) = \mathbf{0}$$

Step 3: Combine like terms

$$\begin{aligned} \mathbf{i} \text{ terms: } & x + 1 - 2 = x - 1 \\ \mathbf{j} \text{ terms: } & 2 + y - 3 = y - 1 \\ \mathbf{k} \text{ terms: } & 3 - 3 + 0 = 0 \end{aligned}$$

Step 4: Set each component to zero

$$x - 1 = 0 \Rightarrow x = 1, \qquad y - 1 = 0 \Rightarrow y = 1$$

Answer: $x = 1,\ y = 1$

Question 8: Show $\overrightarrow{AB}$ is Parallel to $\overrightarrow{CD}$

Given: $A = (1, 2, 1)$, $B = (7, 8, 4)$, $C = (-1, 0, 1)$, $D = (1, 2, 2)$

Step 1: $\overrightarrow{AB} = B - A = (7\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}) - (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$

Step 2: $\overrightarrow{CD} = D - C = (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) - (-\mathbf{i} + \mathbf{k}) = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Step 3: Check if $\overrightarrow{AB}$ is a scalar multiple of $\overrightarrow{CD}$

$$3\overrightarrow{CD} = 3(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} = \overrightarrow{AB}$$

Therefore, $\overrightarrow{AB} = 3\overrightarrow{CD}$, so $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$. ✓

Question 9: Parallel Vectors

(a) Two vectors of length 2 parallel to $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

Step 1: $|\mathbf{v}| = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

Step 2: $\hat{\mathbf{v}} = \dfrac{2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}}{6} = \dfrac{1}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{2}{3}\mathbf{k}$

Step 3: Vectors of length 2: $2\hat{\mathbf{v}}$ and $-2\hat{\mathbf{v}}$

Answer: $\dfrac{2}{3}\mathbf{i} - \dfrac{4}{3}\mathbf{j} + \dfrac{4}{3}\mathbf{k}$ and $-\dfrac{2}{3}\mathbf{i} + \dfrac{4}{3}\mathbf{j} - \dfrac{4}{3}\mathbf{k}$

(b) Find a so that $\mathbf{v} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{w} = a\mathbf{i} + 9\mathbf{j} - 12\mathbf{k}$ are parallel

Step 1: For parallel vectors, $\mathbf{w} = c\,\mathbf{v}$: $\quad a\mathbf{i} + 9\mathbf{j} - 12\mathbf{k} = c\mathbf{i} - 3c\mathbf{j} + 4c\mathbf{k}$

Step 2: Compare components $a = c$; $9 = -3c \Rightarrow c = -3$; $-12 = 4c \Rightarrow c = -3$

Step 3: $a = c = -3$

Answer: $a = -3$

(c) Vector of length 5 opposite to $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

Step 1: $|\mathbf{v}| = \sqrt{1 + 4 + 9} = \sqrt{14}$

Step 2: Unit vector opposite to $\mathbf{v}$: $-\dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{\sqrt{14}}$

Step 3: Vector of length 5: $\dfrac{-5\mathbf{i} + 10\mathbf{j} - 15\mathbf{k}}{\sqrt{14}}$

Answer: $-\dfrac{5}{\sqrt{14}}\mathbf{i} + \dfrac{10}{\sqrt{14}}\mathbf{j} - \dfrac{15}{\sqrt{14}}\mathbf{k}$

(d) Find a and b so that $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k}$ are parallel

Step 1: Let $a\mathbf{i} + b\mathbf{j} - 2\mathbf{k} = c(3\mathbf{i} - \mathbf{j} + 4\mathbf{k}) = 3c\mathbf{i} - c\mathbf{j} + 4c\mathbf{k}$

Step 2: Compare components $a = 3c;\ b = -c;\ -2 = 4c \Rightarrow c = -\dfrac{1}{2}$

Step 3: $a = 3\left(-\dfrac{1}{2}\right) = -\dfrac{3}{2}, \quad b = -\left(-\dfrac{1}{2}\right) = \dfrac{1}{2}$

Answer: $a = -\dfrac{3}{2},\ b = \dfrac{1}{2}$

CHAPTER 5

Displacement and Direction Angles (Questions 10--12)

Question 10: Spacecraft Displacement

Given: From $(120, 240, -50)$ to $(130, 210, 80)$

Step 1: Displacement vector

$$\begin{aligned}\Delta\mathbf{r} &= (130-120)\mathbf{i} + (210-240)\mathbf{j} + \big(80-(-50)\big)\mathbf{k} \\ &= 10\mathbf{i} - 30\mathbf{j} + 130\mathbf{k}\end{aligned}$$

Step 2: Magnitude

$$\begin{aligned}|\Delta\mathbf{r}| &= \sqrt{10^2 + (-30)^2 + 130^2} = \sqrt{100 + 900 + 16900} \\ &= \sqrt{17900} = 10\sqrt{179} \approx 133.79\end{aligned}$$

Answer: $10\sqrt{179}$ km $(\approx 133.79 \text{ km})$

Question 11: Direction Cosines

(i) $\mathbf{u} = -6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

Step 1: $|\mathbf{u}| = \sqrt{36+9+4} = \sqrt{49} = 7$

Step 2: $\cos\alpha = -\frac{6}{7}, \ \cos\beta = \frac{3}{7}, \ \cos\gamma = \frac{2}{7}$

Answer: $\left(-\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$

(ii) $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

Step 1: $|\mathbf{v}| = \sqrt{16+4+25} = \sqrt{45} = 3\sqrt{5}$

Step 2: $\cos\alpha = \dfrac{4}{3\sqrt{5}},\ \cos\beta = \dfrac{2}{3\sqrt{5}},\ \cos\gamma = -\dfrac{5}{3\sqrt{5}}$

Answer: $\left(\dfrac{4}{3\sqrt{5}}, \dfrac{2}{3\sqrt{5}}, -\dfrac{5}{3\sqrt{5}}\right)$

(iii) $\overrightarrow{PQ}$**, where** $P(9,3,13)$ **and** $Q(1,6,19)$

Step 1: $\overrightarrow{PQ} = Q-P = (1-9)\mathbf{i}+(6-3)\mathbf{j}+(19-13)\mathbf{k} = -8\mathbf{i}+3\mathbf{j}+6\mathbf{k}$

Step 2: $|\overrightarrow{PQ}| = \sqrt{64+9+36} = \sqrt{109}$

Step 3: $\cos\alpha = -\dfrac{8}{\sqrt{109}},\ \cos\beta = \dfrac{3}{\sqrt{109}},\ \cos\gamma = \dfrac{6}{\sqrt{109}}$

Answer: $\left(-\dfrac{8}{\sqrt{109}}, \dfrac{3}{\sqrt{109}}, \dfrac{6}{\sqrt{109}}\right)$

Question 12: Direction Angles

Check: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(i) $45^\circ, 45^\circ, 60^\circ$

$$\cos^2 45^\circ + \cos^2 45^\circ + \cos^2 60^\circ = \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = \frac{5}{4} \neq 1$$

Answer: Not possible

(ii) $30^\circ, 45^\circ, 60^\circ$

$$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2} \neq 1$$

Answer: Not possible

(iii) $45^\circ, 60^\circ, 60^\circ$

$$\cos^2 45^\circ + \cos^2 60^\circ + \cos^2 60^\circ = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

Answer: Possible

Summary of Results -- Part II

Questions 7--12

Q.	Answer
7	$x = 1,\ y = 1$
8	$\overrightarrow{AB} = 3\overrightarrow{CD}$ ✓
9(a)	$\frac{2}{3}\mathbf{i} - \frac{4}{3}\mathbf{j} + \frac{4}{3}\mathbf{k},\ -\frac{2}{3}\mathbf{i} + \frac{4}{3}\mathbf{j} - \frac{4}{3}\mathbf{k}$
9(b)	$a = -3$
9(c)	$-\frac{5}{\sqrt{14}}\mathbf{i} + \frac{10}{\sqrt{14}}\mathbf{j} - \frac{15}{\sqrt{14}}\mathbf{k}$
9(d)	$a = -\frac{3}{2},\ b = \frac{1}{2}$
10	$10\sqrt{179}$ km
11(i)	$\left(-\frac{6}{7}, \frac{3}{7}, \frac{2}{7}\right)$
11(ii)	$\left(\frac{4}{3\sqrt{5}}, \frac{2}{3\sqrt{5}}, -\frac{5}{3\sqrt{5}}\right)$
11(iii)	$\left(-\frac{8}{\sqrt{109}}, \frac{3}{\sqrt{109}}, \frac{6}{\sqrt{109}}\right)$
12(i)	Not possible
12(ii)	Not possible
12(iii)	Possible