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Exercise 14.2

Dot Product and Applications

Complete Solutions -- Colorful Booklet Edition

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Key Formula

Cosine of the Angle Between Two Vectors

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$$

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CHAPTER 1

Cosine of the Angle Between Vectors (Question 1)

(i) $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}, \;\; \mathbf{v} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

Step 1: Dot product

$$\mathbf{u} \cdot \mathbf{v} = 2(-1) + 3(2) + 1(2) = -2 + 6 + 2 = 6$$

Step 2: Magnitudes

$$|\mathbf{u}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$
$$|\mathbf{v}| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Step 3:

$$\cos\theta = \frac{6}{3\sqrt{14}} = \frac{2}{\sqrt{14}}$$

Answer: $\cos\theta = \dfrac{2}{\sqrt{14}}$

(ii) $\mathbf{u} = (-3, 2, 5), \;\; \mathbf{v} = (1, 6, -2)$

Step 1: Dot product

$$\mathbf{u} \cdot \mathbf{v} = (-3)(1) + 2(6) + 5(-2) = -3 + 12 - 10 = -1$$

Step 2: Magnitudes

$$|\mathbf{u}| = \sqrt{(-3)^2 + 2^2 + 5^2} = \sqrt{9 + 4 + 25} = \sqrt{38}$$
$$|\mathbf{v}| = \sqrt{1^2 + 6^2 + (-2)^2} = \sqrt{1 + 36 + 4} = \sqrt{41}$$

Step 3:

$$\cos\theta = \frac{-1}{\sqrt{38}\sqrt{41}} = -\frac{1}{\sqrt{1558}}$$

Answer: $\cos\theta = -\dfrac{1}{\sqrt{1558}}$

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CHAPTER 2

Angles from Given Magnitudes (Questions 2--3)

Question 2: $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}, \; |\mathbf{a}| = 3, \; |\mathbf{b}| = 5, \; |\mathbf{c}| = 7$

Step 1: From $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, we have $\mathbf{c} = -(\mathbf{a} + \mathbf{b})$.
Step 2: **Square both sides** – $|\mathbf{c}|^2 = |\mathbf{a} + \mathbf{b}|^2$

$$\begin{aligned} 49 &= |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2(\mathbf{a} \cdot \mathbf{b}) \\ 49 &= 9 + 25 + 2(\mathbf{a} \cdot \mathbf{b}) \\ 49 &= 34 + 2(\mathbf{a} \cdot \mathbf{b}) \\ 2(\mathbf{a} \cdot \mathbf{b}) &= 15 \;\Rightarrow\; \mathbf{a} \cdot \mathbf{b} = \frac{15}{2} \end{aligned}$$

Step 3:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{15/2}{3 \times 5} = \frac{15/2}{15} = \frac{1}{2}$$

Step 4: $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

Answer: 60°

Question 3: $|\mathbf{a}| = 3, \; |\mathbf{b}| = 4, \; |\mathbf{a} + \mathbf{b}| = 5$

Step 1:

$$\begin{aligned} |\mathbf{a} + \mathbf{b}|^2 &= |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2(\mathbf{a} \cdot \mathbf{b}) \\ 25 &= 9 + 16 + 2(\mathbf{a} \cdot \mathbf{b}) \\ 25 &= 25 + 2(\mathbf{a} \cdot \mathbf{b}) \;\Rightarrow\; \mathbf{a} \cdot \mathbf{b} = 0 \end{aligned}$$

Step 2: $\cos\theta = \dfrac{0}{3 \times 4} = 0$

Step 3: $\theta = 90^\circ$
Answer: 90°

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CHAPTER 3

Projections and Perpendicularity (Questions 4--6)

Question 4: Projections

Formulas: Projection of **a** along **b**: $\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|}$ Projection of **b** along **a**: $\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

(i) $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

Step 1: $\mathbf{a}\cdot\mathbf{b} = 2(1) + 3(-2) + (-1)(4) = 2 - 6 - 4 = -8$

Step 2: $|\mathbf{a}| = \sqrt{4+9+1} = \sqrt{14}$, $|\mathbf{b}| = \sqrt{1+4+16} = \sqrt{21}$

Step 3: Projection of **a** along **b**: $-\frac{8}{\sqrt{21}}$ Projection of **b** along **a**: $-\frac{8}{\sqrt{14}}$

Answer: $-\frac{8}{\sqrt{21}}$ and $-\frac{8}{\sqrt{14}}$

(ii) $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

Step 1: $\mathbf{a}\cdot\mathbf{b} = 4(1) + (-2)(1) + 3(1) = 4 - 2 + 3 = 5$

Step 2: $|\mathbf{a}| = \sqrt{16+4+9} = \sqrt{29}$, $|\mathbf{b}| = \sqrt{1+1+1} = \sqrt{3}$

Step 3: Projection of **a** along **b**: $\frac{5}{\sqrt{3}}$ Projection of **b** along **a**: $\frac{5}{\sqrt{29}}$

Answer: $\frac{5}{\sqrt{3}}$ and $\frac{5}{\sqrt{29}}$

Question 5: Find α so the Vectors are Perpendicular

Condition: $\mathbf{u}\cdot\mathbf{v} = 0$

(i) $\mathbf{u} = \alpha\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \alpha\mathbf{k}$

Step 1: $\mathbf{u}\cdot\mathbf{v} = \alpha(1) + 3(-2) + 1(\alpha) = 2\alpha - 6$
Step 2: $2\alpha - 6 = 0 \;\Rightarrow\; \alpha = 3$
Answer: $\alpha = 3$

(ii) $\mathbf{u} = \alpha\mathbf{i} + 2\alpha\mathbf{j} - \mathbf{k},\ \mathbf{v} = \mathbf{i} + \alpha\mathbf{j} + 3\mathbf{k}$
Step 1: $\mathbf{u}\cdot\mathbf{v} = \alpha(1) + (2\alpha)(\alpha) + (-1)(3) = 2\alpha^2 + \alpha - 3$
Step 2: $2\alpha^2 + \alpha - 3 = 0 \;\Rightarrow\; (2\alpha+3)(\alpha-1) = 0$
Step 3: $\alpha = 1$ or $\alpha = -\frac{3}{2}$
Answer: $\alpha = 1$ or $\alpha = -\frac{3}{2}$

Question 6: Right Triangle at C

Given: $A(3,0,-2)$, $B(0,3,1)$, $C(1,1,z)$
Step 1: For a right angle at C: $\overrightarrow{CA}\cdot\overrightarrow{CB} = 0$
Step 2: $\overrightarrow{CA} = A - C = 2\mathbf{i} - \mathbf{j} - (z+2)\mathbf{k}$
Step 3: $\overrightarrow{CB} = B - C = -\mathbf{i} + 2\mathbf{j} + (1-z)\mathbf{k}$
Step 4: Dot product

$$\overrightarrow{CA}\cdot\overrightarrow{CB} = 2(-1)+(-1)(2)+\left[-(z+2)\right](1-z) = -4-(z+2)(1-z)$$

Step 5: Expand $(z+2)(1-z) = z + 2 - z^2 - 2z = -z^2 - z + 2$
Step 6: Dot product

$$\overrightarrow{CA}\cdot\overrightarrow{CB} = -4 - (-z^2 - z + 2) = z^2 + z - 6$$

Step 7: Set equal to zero

$$z^2 + z - 6 = 0 \;\Rightarrow\; (z+3)(z-2) = 0 \;\Rightarrow\; z = -3 \text{ or } z = 2$$

Answer: $z = -3$ or $z = 2$

Summary of Results -- Part I

Questions 1--6

Q.	Answer
1(i)	$\cos\theta = \dfrac{2}{\sqrt{14}}$
1(ii)	$\cos\theta = -\dfrac{1}{\sqrt{1558}}$
2	60°
3	90°
4(i)	$-\dfrac{8}{\sqrt{21}},\ -\dfrac{8}{\sqrt{14}}$
4(ii)	$\dfrac{5}{\sqrt{3}},\ \dfrac{5}{\sqrt{29}}$
5(i)	$\alpha = 3$
5(ii)	$\alpha = 1$ or $-\dfrac{3}{2}$
6	$z = -3$ or $z = 2$

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CHAPTER 4

Vector Identities and Proofs (Questions 7--12)

Question 7: Show $\sin\theta = \frac{1}{2}|\mathbf{a}-\mathbf{b}|$

Given: $\mathbf{a}, \mathbf{b}$ are unit vectors and 2θ is the angle between them.

Step 1: $|\mathbf{a}-\mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\,\mathbf{a}\cdot\mathbf{b}$

Step 2: Since $\mathbf{a}, \mathbf{b}$ are unit vectors, $|\mathbf{a}| = |\mathbf{b}| = 1$ and $\mathbf{a}\cdot\mathbf{b} = \cos 2\theta$:

$$|\mathbf{a}-\mathbf{b}|^2 = 1 + 1 - 2\cos 2\theta = 2 - 2\cos 2\theta$$

Step 3: $2 - 2\cos 2\theta = 2(1-\cos 2\theta) = 2\big(2\sin^2\theta\big) = 4\sin^2\theta$

Step 4: $|\mathbf{a}-\mathbf{b}| = 2\sin\theta$ (since $\theta \in [0, \pi/2]$, $\sin\theta \geq 0$)

Step 5: $\sin\theta = \dfrac{1}{2}|\mathbf{a}-\mathbf{b}|$ ✓

Question 8: If $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$, Show $\mathbf{a}\perp\mathbf{b}$

Step 1: Square both sides $|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}-\mathbf{b}|^2$

Step 2: Expand

$$|\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\,\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\,\mathbf{a}\cdot\mathbf{b}$$

Step 3: Cancel $|\mathbf{a}|^2 + |\mathbf{b}|^2$

$$2\,\mathbf{a}\cdot\mathbf{b} = -2\,\mathbf{a}\cdot\mathbf{b} \;\Rightarrow\; 4\,\mathbf{a}\cdot\mathbf{b} = 0 \;\Rightarrow\; \mathbf{a}\cdot\mathbf{b} = 0$$

Therefore, **a** and **b** are perpendicular. ✓

Question 9: Right-Triangle Verification

(i) Vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{c} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

Step 1: Check whether they close a triangle $\mathbf{a}+\mathbf{b}+\mathbf{c} = 6\mathbf{i}-4\mathbf{j}+2\mathbf{k} \neq \mathbf{0}$, so these are not the three sides of a triangle placed head-to-tail; instead we treat them as three vectors and test for a right angle among them.

Step 2: Dot products

$$\mathbf{a} \cdot \mathbf{b} = 3(1) + (-2)(-3) + 1(5) = 14 \neq 0$$
$$\mathbf{b} \cdot \mathbf{c} = 1(2) + (-3)(1) + 5(-4) = -21 \neq 0$$
$$\mathbf{a} \cdot \mathbf{c} = 3(2) + (-2)(1) + 1(-4) = 0$$

Since $\mathbf{a} \cdot \mathbf{c} = 0$, vectors $\mathbf{a}$ and $\mathbf{c}$ are perpendicular.

Answer: A right angle exists between $\mathbf{a}$ and $\mathbf{c}$.

(ii) Points $P(4, -1, 2)$, $Q(1, 3, -1)$, $R(-2, 4, 6)$

Step 1: Side vectors

$$\overrightarrow{PQ} = -3\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$$
$$\overrightarrow{PR} = -6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$$
$$\overrightarrow{QR} = -3\mathbf{i} + \mathbf{j} + 7\mathbf{k}$$

Step 2: Dot products

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 18 + 20 - 12 = 26 \neq 0$$
$$\overrightarrow{PQ} \cdot \overrightarrow{QR} = 9 + 4 - 21 = -8 \neq 0$$
$$\overrightarrow{PR} \cdot \overrightarrow{QR} = 18 + 5 + 28 = 51 \neq 0$$

Answer: None are perpendicular, so $\triangle PQR$ is *not* a right triangle.

Question 10: Prove $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Step 1: Consider the unit vectors $\mathbf{u} = \cos\alpha\,\mathbf{i} - \sin\alpha\,\mathbf{j}$ and $\mathbf{v} = \cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j}$, at angles $-\alpha$ and β respectively, so the angle between them is $\alpha+\beta$.

Step 2: Dot product

$$\mathbf{u}\cdot\mathbf{v} = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

Step 3: Since $\mathbf{u}, \mathbf{v}$ are unit vectors, $\mathbf{u}\cdot\mathbf{v} = \cos(\alpha+\beta)$, so

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \checkmark$$

Question 11: Triangle ABC with Side Vectors

Note

Part (i) as originally stated, $\mathbf{b} = \cos\alpha\,\mathbf{a} + \cos\beta\,\mathbf{b}$, contains a typo. The intended identity is part (ii) below.

(ii) $c = \cos\alpha\, b + \cos\beta\, a$ – In triangle ABC, side c (opposite angle C) equals the sum of the projections of sides a and b onto it.
(iii) $b^2 = a^2 + c^2 - 2ac\cos\beta$ – the Law of Cosines, where β is the angle between sides a and c (opposite side b).
(iv) $c^2 = a^2 + b^2 - 2ab\cos\alpha$ – the Law of Cosines, where α is the angle between sides a and b (opposite side c). $\checkmark$

Question 12: Show $|\mathbf{a}-\mathbf{b}| \le |\mathbf{a}+\mathbf{b}| \le |\mathbf{a}| + |\mathbf{b}|$

Step 1:

$$|\mathbf{a}-\mathbf{b}|^2 - |\mathbf{a}+\mathbf{b}|^2 = \left(|\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\,\mathbf{a}\cdot\mathbf{b}\right) - \left(|\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\,\mathbf{a}\cdot\mathbf{b}\right) = -4\,\mathbf{a}\cdot\mathbf{b}$$

Note

This difference is ≤ 0 (so that $|\mathbf{a}-\mathbf{b}| \leq |\mathbf{a}+\mathbf{b}|$) only when $\mathbf{a}\cdot\mathbf{b} \geq 0$. The inequality as originally stated is therefore not true in general. The correct, always-true statement is the triangle inequality:

$$\big||\mathbf{a}|-|\mathbf{b}|\big| \leq |\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}|+|\mathbf{b}|$$

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CHAPTER 5

Work Done by a Force (Questions 13--16)

Question 13: Work Done

Given: $\mathbf{F} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$, from $P_1(2, -3, 1)$ to $P_2(7, 5, 3)$
Step 1: Displacement

$$\mathbf{d} = (7-2)\mathbf{i} + (5-(-3))\mathbf{j} + (3-1)\mathbf{k} = 5\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}$$

Step 2: Work $= \mathbf{F} \cdot \mathbf{d} = 2(5) + 5(8) + 3(2) = 10 + 40 + 6 = 56$
Answer: 56 units

Question 14: Work Done by Two Forces

Given: $\mathbf{F}_1 = 3\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, $\mathbf{F}_2 = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$, from $A(2, 1, 3)$ to $B(5, 4, 4)$
Step 1: Resultant force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = 4\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$
Step 2: Displacement $\mathbf{d} = (5-2)\mathbf{i} + (4-1)\mathbf{j} + (4-3)\mathbf{k} = 3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
Step 3: Work $= \mathbf{F} \cdot \mathbf{d} = 4(3) + 8(3) + (-4)(1) = 12 + 24 - 4 = 32$
Answer: 32 units

Question 15: Total Work Done by Three Forces

Given: $\mathbf{F}_1 = 10\mathbf{i} - \mathbf{j} + 11\mathbf{k}$, $\mathbf{F}_2 = 4\mathbf{i} + 5\mathbf{j} + 9\mathbf{k}$, $\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} - 9\mathbf{k}$, from $A(5, -5, -7)$ to $B(6, 2, -2)$
Step 1: Resultant force

$$\mathbf{F} = (10 + 4 - 2)\mathbf{i} + (-1 + 5 + 1)\mathbf{j} + (11 + 9 - 9)\mathbf{k} = 12\mathbf{i} + 5\mathbf{j} + 11\mathbf{k}$$

Step 2: Displacement $\mathbf{d} = (6-5)\mathbf{i} + (2-(-5))\mathbf{j} + (-2-(-7))\mathbf{k} = \mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$
Step 3: Work $= \mathbf{F} \cdot \mathbf{d} = 12(1) + 5(7) + 11(5) = 12 + 35 + 55 = 102$

Answer: 102 units ✓

Question 16: Work Done -- Force of Given Magnitude

Given: Force of magnitude 6, parallel to $4\mathbf{i}+3\mathbf{j}-\mathbf{k}$, from $A(2,-1,3)$ to $B(7,3,2)$

Step 1: Unit vector in the direction of the force

$$|4\mathbf{i}+3\mathbf{j}-\mathbf{k}| = \sqrt{16+9+1} = \sqrt{26}, \qquad \hat{\mathbf{F}} = \frac{4\mathbf{i}+3\mathbf{j}-\mathbf{k}}{\sqrt{26}}$$

Step 2: Force vector

$$\mathbf{F} = 6\hat{\mathbf{F}} = \frac{24\mathbf{i}+18\mathbf{j}-6\mathbf{k}}{\sqrt{26}}$$

Step 3: Displacement $\mathbf{d} = (7-2)\mathbf{i}+(3-(-1))\mathbf{j}+(2-3)\mathbf{k} = 5\mathbf{i}+4\mathbf{j}-\mathbf{k}$

Step 4: Work

$$\begin{aligned} \mathbf{F}\cdot\mathbf{d} &= \frac{24(5)+18(4)+(-6)(-1)}{\sqrt{26}} \\ &= \frac{120+72+6}{\sqrt{26}} = \frac{198}{\sqrt{26}} \end{aligned}$$

Answer: $\dfrac{198}{\sqrt{26}}$ units

Summary of Results -- Part II

Questions 7--16

Q.	Answer
7	Proven ✓
8	Proven ✓
9(i)	Right angle between **a** and **c**
9(ii)	Not a right triangle
10	Proven ✓
11	Typo in (i); (ii)–(iv) are Law of Cosines identities
12	Statement corrected – see triangle inequality
13	56 units
14	32 units
15	102 units ✓
16	$\frac{198}{\sqrt{26}}$ units

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