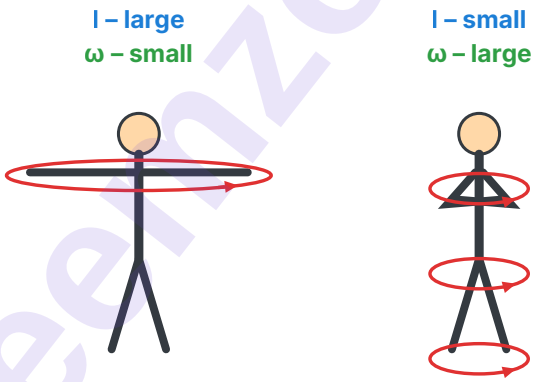
PHYSICS · CLASS 11 · NEW SYLLABUS

UNIT 3

Circular and Rotational Motion

Complete Study Booklet



Long Questions | Short Questions | MCQs | Solved Exercise

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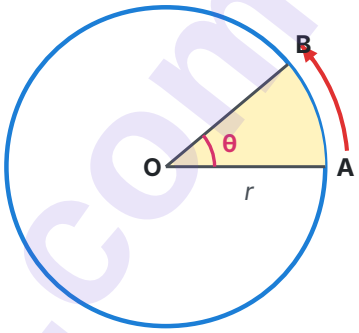
PART A

Long Questions

Q1 **Define angular displacement. How is its direction found by the right-hand rule? Establish S = rθ and show that 1 radian = 57.3°.**

Angular displacement is the angle subtended by an arc at the centre of the circle. It is denoted by θ.

For **small** values θ is a vector quantity; for **large** values it is not a vector.

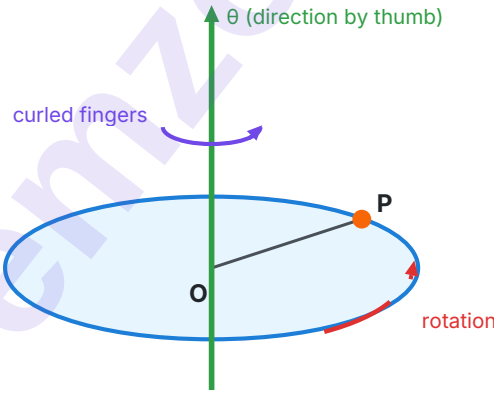


Angular displacement θ

Right-Hand Rule

Grasp the axis of rotation in the right hand with the fingers curled in the direction of rotation. The extended thumb along the axis gives the direction of angular displacement.

Units: degree, revolution or radian; the **radian** is the SI unit.



Right-hand rule: fingers along rotation, thumb gives direction

Radian

One radian is the angle subtended at the centre of a circle by an arc whose length equals the radius of the circle.

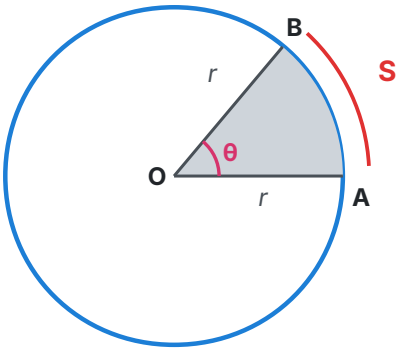
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Relation S = rθ

For an arc of length S on a circle of radius r subtending angle θ, at constant radius the arc length is directly proportional to the angle:

$$S \propto \theta \Rightarrow S = r\theta$$

$$\boldsymbol{\theta = \frac{S}{r}\ \text{(rad)} \ldots(1)}$$



θ = S / r (1 rad when S = r)

Arc S subtends θ at the centre

Radian and degree

For one complete revolution, S = 2πr and θ = 360°. Equation (1) gives:

$$360° = \frac{2\pi r}{r}\ \text{rad} = 2\pi\ \text{rad}$$

$$\boldsymbol{1\ \text{rad} = \frac{360°}{2\pi} = \frac{180°}{\pi} = 57.3° \quad \text{or} \quad 1° = 0.01745\ \text{rad}}$$

Q2 Define angular velocity. Establish the relation v = rω.

Angular velocity is the time rate of change of angular displacement.

Average angular velocity — total change in angular displacement divided by total time:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity — angular velocity at any instant:

$$\omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

It is a vector; its direction is given by the right-hand rule. **SI unit:** rad s^{-1} (also rev min^{-1}, rpm).

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Relation between linear and angular velocity

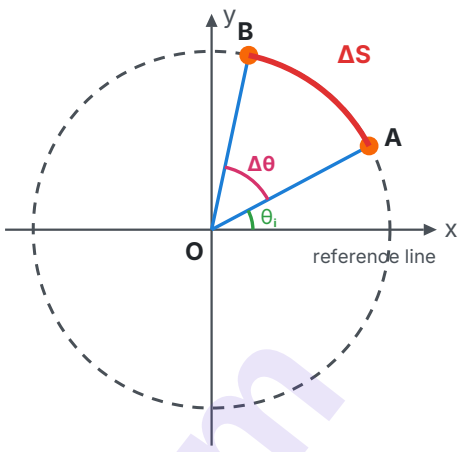
A body moves on a circle of radius r from A to B in time Δt, covering arc ΔS with angular displacement Δθ:

$$\Delta S = r\,\Delta\theta$$

Dividing by Δt and taking the limit Δt → 0:

$$\lim_{\Delta t\to 0}\frac{\Delta S}{\Delta t} = r\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$$

The left side is the tangential velocity v, and the limit on the right is ω:



Particle moving from A to B

$$\mathbf{v = r\omega}$$

In vector form: $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$.

Q3 Define angular acceleration. Establish the relation a = rα.

Angular acceleration (α) is the rate of change of angular velocity with time.

Average angular acceleration:

$$\boldsymbol{\alpha}_{av} = \frac{\Delta\boldsymbol{\omega}}{\Delta t} = \frac{\boldsymbol{\omega}_f - \boldsymbol{\omega}_i}{t_f - t_i}$$

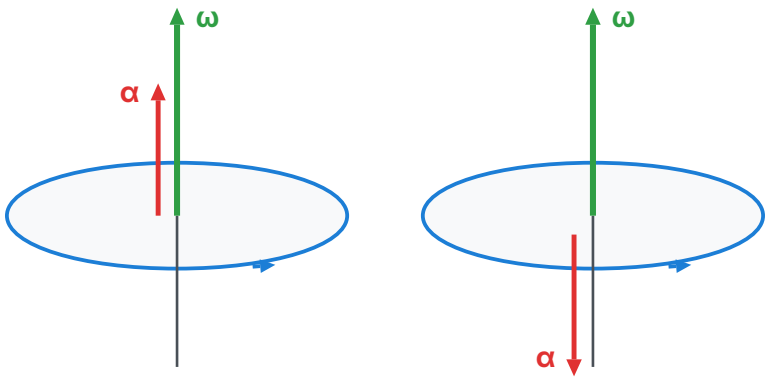
It is a vector along $\Delta\boldsymbol{\omega}$. Units: deg s^{-2}, rev s^{-2} or **rad s^{-2}** (SI).

Instantaneous angular acceleration — rate of change of angular velocity at any instant:

$$\boldsymbol{\alpha}_{inst} = \lim_{\Delta t\to 0}\frac{\Delta\boldsymbol{\omega}}{\Delta t}$$

For rotation in a fixed plane, α points along the axis of rotation: **parallel** to ω if ω is increasing, **anti-parallel** if ω is decreasing.

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ω increasing: α parallel to ω ω decreasing: α anti-parallel to ω

Direction of α relative to ω

Relation between tangential and angular acceleration

A body moves on a circle of radius r; its angular velocity changes from ω_i at A to ω_f at B in time Δt. Using $v = r\omega$:

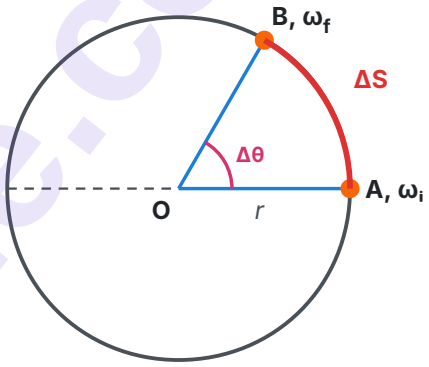
$$v_i = r\omega_i \ldots(1) \quad v_f = r\omega_f \ldots(2)$$

Subtracting (1) from (2):

$$\Delta v = r(\omega_f - \omega_i) = r\,\Delta\omega$$

Dividing by Δt and taking the limit Δt → 0:

$$\lim_{\Delta t\to 0} \frac{\Delta v}{\Delta t} = r \lim_{\Delta t\to 0} \frac{\Delta\omega}{\Delta t}$$



Angular velocity changing from ω_i to ω_f

$a = r\alpha$ (vector form: $a = \alpha \times r$)

Q4 Give a comparison of linear and angular equations of motion.

For motion with **constant angular acceleration**, the angular equations follow from the linear ones by replacing $v_i \to \omega_i$, $S \to \theta$, $v_f \to \omega_f$ and $a \to \alpha$.

#	Angular motion	Linear motion
1	$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
2	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$S = v_i t + \frac{1}{2}at^2$
3	$2\alpha\theta = \omega_f^2 - \omega_i^2$	$2aS = v_f^2 - v_i^2$

The angular equations hold only when the **axis of rotation is fixed**.

Q5 Define and explain centripetal force. Give applications of centripetal force.

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The force needed to bend the straight path of a particle into a circular path is called the **centripetal force**.

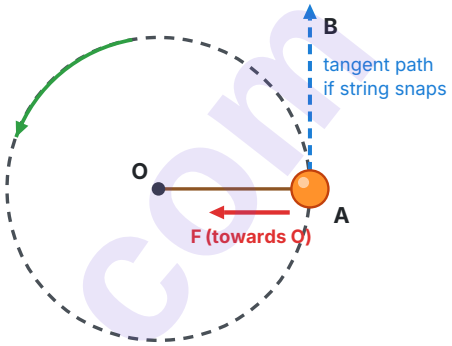
$$F_c = ma_c = \frac{mv^2}{r} \quad ...(1) \qquad F_c = mr\omega^2 \quad ...(2)$$

where $a_c = v^2/r$ is the centripetal acceleration, directed towards the centre (using $v = r\omega$).

Explanation

A force along the velocity changes only the speed. A constant force **perpendicular** to the velocity changes only the direction, keeping speed constant. Such a force makes a body move in a circle; it is the centripetal (centre-seeking) force.

A ball whirled on a string keeps moving in a circle. If the string snaps at A, the ball flies off along the **tangent** AB. So a force along the radius, towards the centre and perpendicular to the velocity, is needed at every point.



Ball on a string; tangent path if the string snaps

Examples (who supplies the centripetal force?)

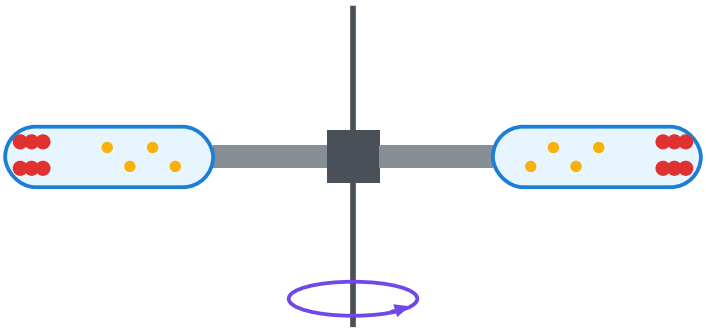
1. Ball whirled on a string — **tension** in the string.
2. Object on a turntable — **friction**.
3. Earth around the Sun, Moon and satellites around the Earth — **gravitational force**.
4. Charged particle in a magnetic field — **magnetic force** perpendicular to its motion.
5. Vehicle on a turn — **friction** between tyres and road. On a slippery road at high speed friction may be insufficient and the vehicle may skid or topple. So highways are **banked**: the outer edge is raised above the inner edge.

Applications

If the available force falls short of the required centripetal force, the object moves away from the centre. This is the principle of the centrifuge.

1. **Centrifuge:** a laboratory device that separates denser and lighter particles. The mixture is spun fast; denser particles settle at the bottom (outer end) of the sample tubes and lighter ones rise to the top.

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denser particles (red) move to the outer ends

Centrifuge: sample tubes spinning about a vertical axis

1. **Washing-machine dryer:** a perforated cylinder spins rapidly; water moves outward to the wall and drains through the holes, so clothes dry quickly.
2. **Cream separator:** milk is whirled rapidly. Light particles (cream) gather near the axis, while heavier particles move outward, so cream is easily separated.

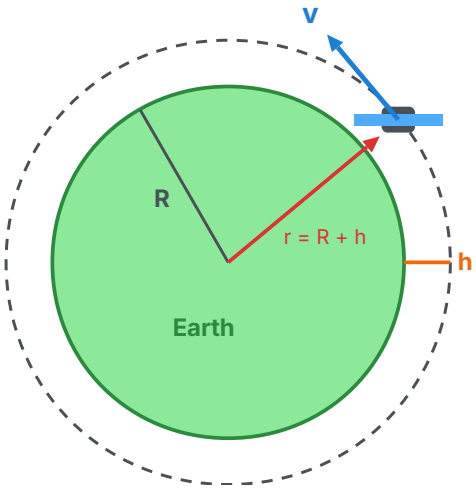
Q6 What is an artificial satellite? Find the expression for the orbital speed of a satellite orbiting very close to the Earth and evaluate it.

Artificial satellites are man-made objects orbiting the Earth.

They are put into orbit by rockets. Gravity then keeps them orbiting; the gravitational force between the Earth and the satellite supplies the centripetal force:

$$F_c = \frac{mv^2}{r}$$

m = mass of satellite, v = orbital speed, r = radius of orbit.



Satellite at height h above the Earth

Minimum (critical) velocity

For a satellite close to the Earth, h ≪ R, so r = R + h ≈ R:

$$F_c = F_g \Rightarrow \frac{mv^2}{R} = mg \Rightarrow v^2 = gR \Rightarrow v = \sqrt{(gR)}$$

$$\mathbf{v = \sqrt{(6.4 \times 10^6 \times 9.8)} = 7.9 \times 10^3\ m\ s^{-1} = 7.9\ km\ s^{-1}}$$

This is the minimum velocity needed to put a satellite into orbit near the Earth, called the **critical velocity**.

Time period

Time to complete one revolution: with s = 2πR at constant speed,

$$T = \frac{2\pi R}{v} = \frac{2 \times 3.14 \times 6.4 \times 10^6}{7.9 \times 10^3} \approx 5090 \text{ s} \approx 84 \text{ min}$$

The higher the satellite, the slower its required speed and the longer its period.

Point to Ponder

Why don't you fall out of your seat when a roller coaster goes upside down?
At the top of the loop, gravity and the seat's push together supply the centripetal force needed to keep you on the circular path. At enough speed, more force is needed than gravity alone gives, so you are pressed into the seat and do not fall.

Q7 Define orbital velocity. Derive an expression for the orbital speed of a satellite. How does it depend on orbital radius?

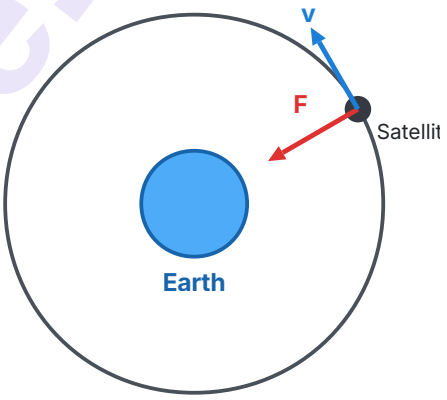
Orbital velocity is the tangential velocity of a satellite orbiting the Earth.

A satellite of mass m_s moves in an orbit of radius r with speed v. The gravitational force supplies the centripetal force:

$$F_c = F_g \Rightarrow \frac{m_s v^2}{r} = \frac{G m_s M}{r^2}$$

where M is the mass of the Earth:

$$v^2 = \frac{GM}{r} \Rightarrow v = \sqrt{\left(\frac{GM}{r}\right)} \ldots(1)$$



Gravity supplies the centripetal force

Since √(GM) is constant:

$$v \propto \frac{1}{\sqrt{r}}$$

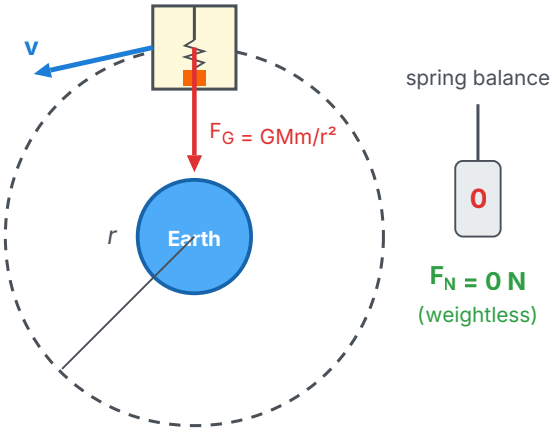
Orbital velocity is **inversely proportional to the square root of the orbital radius**. If the satellite's speed is less than that given by (1), it falls back to the Earth.

Q8 What is meant by weightlessness? Show quantitatively that satellites orbiting the Earth are in a state of weightlessness.

Weightlessness is the state in which the apparent weight of an object becomes zero.

Everything inside an orbiting satellite feels weightless because the satellite is accelerating towards the Earth's centre like a freely falling body.

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A body hanging from a spring balance in an orbiting satellite

A body of mass m hangs from a spring balance inside a satellite of mass M orbiting at radius r. Two forces act on it: its weight mg (down) and the supporting force F_N (tension, up). Their resultant provides the centripetal force:

$$F_c = mg - F_N \Rightarrow \frac{mv^2}{r} = mg - F_N \quad ...(2)$$

For the satellite itself, gravity supplies the centripetal force:

$$Mg = \frac{Mv^2}{r} \Rightarrow g = \frac{v^2}{r}$$

So $mv^2/r = mg$, and equation (2) becomes:

$$\mathbf{mg = mg - F_N \Rightarrow F_N = 0}$$

The supporting force on a body inside the satellite is zero, so bodies and astronauts in a satellite are in a state of **apparent weightlessness**.

Q9 **What is meant by artificial gravity? Derive the relation for the frequency of spin of a satellite to produce artificial gravity.**

Weightlessness is a serious handicap for astronauts doing research. To overcome it, **artificial gravity** is created in the spaceship so astronauts can work as they do under the Earth's gravity.

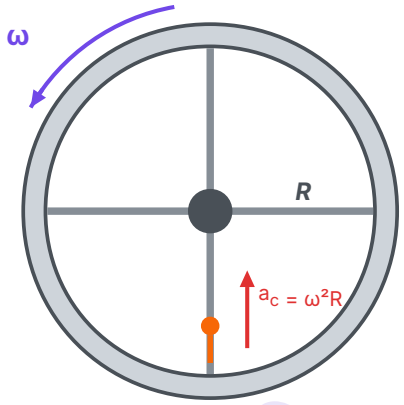
How it is produced: by spinning the spaceship about its own axis. Objects and astronauts are pressed against the outer wall and push on the floor, just as our feet press on the ground on Earth.

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Frequency of spin

The space station is a large hollow ring of average radius R spinning about a vertical axis with speed v = ωR. The centripetal acceleration of an astronaut is:

$$a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R \Rightarrow \omega = \sqrt{\left(\frac{a_c}{R}\right)} \ldots(1)$$



Spinning space station

With ω = 2π/T, where T is the time for one rotation:

$$\frac{2\pi}{T} = \sqrt{\left(\frac{a_c}{R}\right)} \Rightarrow \frac{1}{T} = \frac{1}{2\pi}\sqrt{\left(\frac{a_c}{R}\right)}$$

Since frequency f = 1/T:

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{a_c}{R}\right)}$$

For Earth-like gravity, put $a_c = g$:

$$\mathbf{f = \frac{1}{2\pi}\sqrt{\left(\frac{g}{R}\right)}}$$

At this frequency, astronauts and objects have an apparent weight equal to their real weight.

Q10 **Define and explain the moment of inertia of a rigid body and its significance. Find the expression for the moment of inertia of a rigid body.**

Moment of inertia is the resistance of a body to any change in its state of rest or of uniform circular motion.

Inertia applies to straight-line motion; **moment of inertia** plays the same role in circular motion that mass plays in linear motion. It equals the mass of a particle times the square of its distance from the axis of rotation:

$$\mathbf{I = mr^2} \quad \textbf{SI unit: kg m}^2 \quad \textbf{Dimensions: [ML}^2\textbf{]}$$

It depends on (i) the **mass** of the body and (ii) the **distribution of mass** about the axis of rotation.

Moment of inertia of a particle

A mass m is fixed to a massless rod of length r pivoted at O. A force F acts perpendicular to the rod. By Newton's 2nd law, with $a_t = r\alpha$:

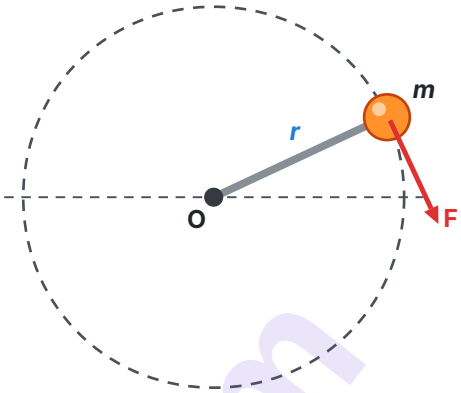
$$F = ma_t = mr\alpha \quad ...(2)$$

Multiplying by r:

$$rF = mr^2\alpha$$

Since $rF = \tau$ (torque) and $mr^2 = I$:

$$\boldsymbol{\tau = I\alpha}$$



Mass on a rod pivoted at O

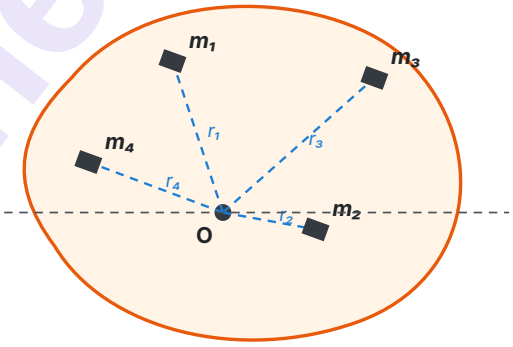
Moment of inertia of a rigid body

A **rigid body** is one in which every particle rotates with the same angular velocity ω and the same angular acceleration α.

Divide the body into small particles $m_1, m_2, ..., m_n$ at distances $r_1, r_2, ..., r_n$ from the axis through O. Torque on each particle:

$$\tau_1 = m_1 r_1^2 \alpha_1, \quad \tau_2 = m_2 r_2^2 \alpha_2, \ ..., \quad \tau_n = m_n r_n^2 \alpha_n$$

Total torque $\tau = \tau_1 + \tau_2 + ... + \tau_n$. For a rigid body $\alpha_1 = \alpha_2 = ... = \alpha$:



$I = \Sigma m_i r_i^2$

Rigid body made of particles

$$\tau = (m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2)\,\alpha = (\Sigma\, m_i r_i^2)\,\alpha$$

$$\boldsymbol{I = \Sigma\, m_i r_i^2 \quad \Rightarrow \quad \tau = I\alpha}$$

Torque on a rigid body equals its moment of inertia times its angular acceleration.

Q11 Define and explain the term angular momentum.

A particle has **angular momentum** about a reference axis if it moves so that its angular position changes relative to that axis.

For a particle of mass m with momentum $\mathbf{p}$ and position vector $\mathbf{r}$ about O:

$$\boldsymbol{L = r \times p}$$

(i) **Spin** angular momentum is due to spin motion; (ii) **orbital** angular momentum is due to orbital motion.

Angular momentum of a particle

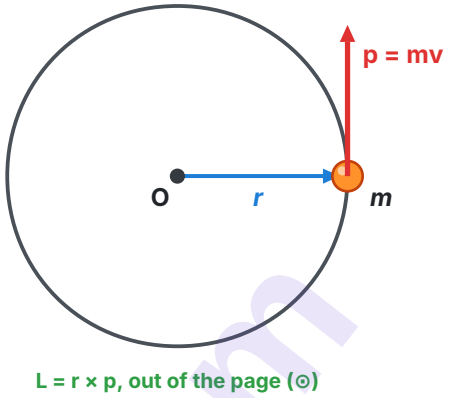
$$L = rp \sin\theta, \quad p = mv$$

For circular motion θ = 90°, sin 90° = 1:

$$L = mvr = mr^2\omega \quad (v = r\omega)$$

Since $mr^2 = I$:

$$\mathbf{L = I\omega}$$



L = r × p, out of the page (⊙)

Particle in circular motion

SI unit: kg m^2 s^{-1} or J s; **dimensions** $[ML^2T^{-1}]$. **Direction:** perpendicular to the plane of **r** and **p**, by the right-hand rule.

Angular momentum of a rigid body

A rigid body rotating about a fixed axis is made of particles m_1, m_2, …, m_n at distances r_1, r_2, …, r_n, all with the same ω:

$$L = L_1 + L_2 + \ldots + L_n = m_1r_1^2\omega + m_2r_2^2\omega + \ldots + m_nr_n^2\omega$$

$$\mathbf{L = (\Sigma\, m_ir_i^2)\,\omega = I\omega}$$

Q12 **State and explain the law of conservation of angular momentum and write a few of its applications in sports.**

In the absence of an external torque, the total angular momentum of a system remains constant.

$$\mathbf{L_{total} = L_1 + L_2 + \ldots + L_n = constant \quad or \quad I\omega = constant \Rightarrow I_1\omega_1 = I_2\omega_2}$$

If an isolated system changes its moment of inertia, its angular velocity changes so that Iω stays the same. Angular momentum is a vector along the axis of rotation, so with no external torque its direction (the axis) also stays fixed:

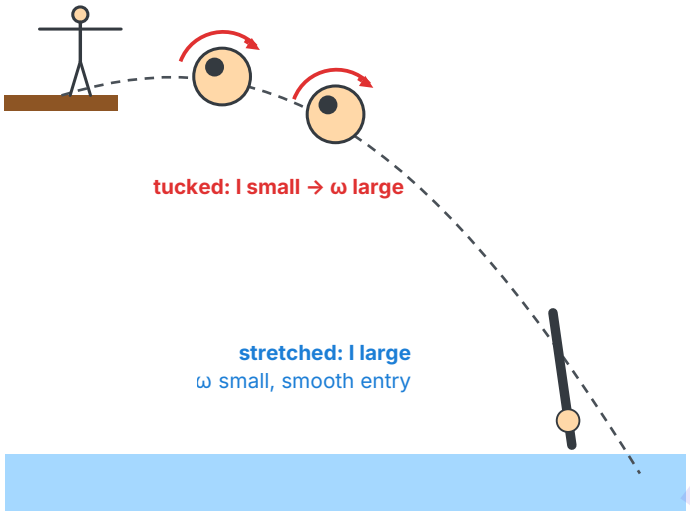
The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

This is why the Earth's axis stays fixed in direction as it orbits the Sun — no sizeable torque acts on it.

Examples

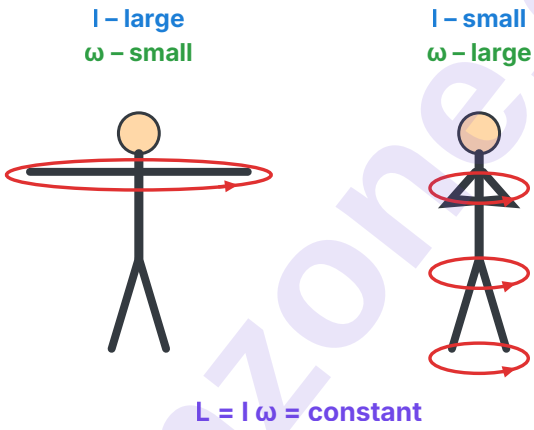
i. Diver from a diving board: after leaving the springboard, he curls his body (arms and legs in). His moment of inertia decreases, so he spins with a large angular velocity and makes somersaults. Before reaching the water he stretches out, increasing I and reducing ω, for a smooth entry.

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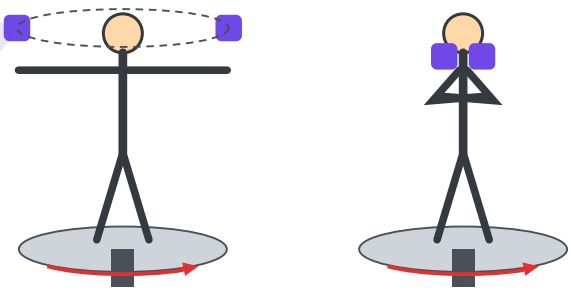
A diver using conservation of angular momentum

ii. Spinning ice skater: folding the arms and bringing the stretched leg close decreases I, so ω increases. Stretching the arms and leg out increases I, so ω decreases.



Ice skater: L = Iω stays constant

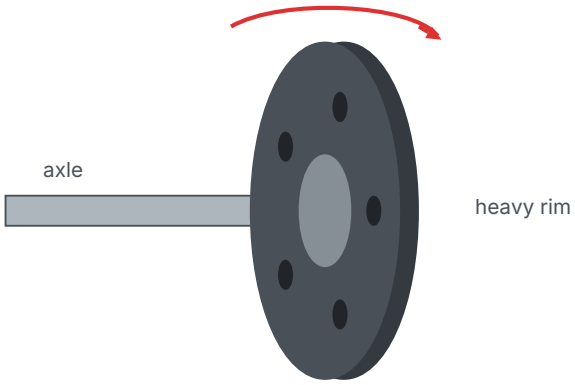
iii. Person on a turntable holding weights: with the dumb-bells stretched out, I is large and the spin is slow. Drawing the hands inward reduces I, so the angular speed increases at once.



(a) arms out: ω small (b) arms in: ω large

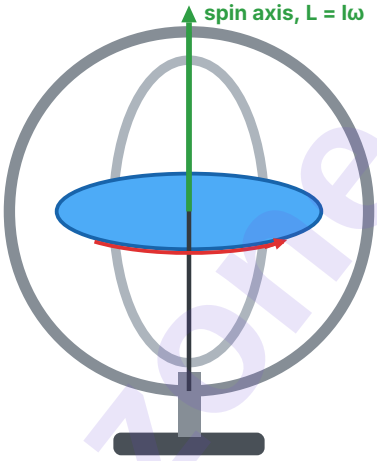
Man with masses on a turntable

iv. Flywheel: a heavy wheel on an axle that stores rotational energy, smooths out output fluctuations and gives stability (bicycles, vehicles, machinery, gyroscopes, ships, spacecraft). Its angular momentum resists changes in orientation.



A flywheel

v. Gyroscope: a mounted flywheel pivoted in supporting rings that keeps its orientation. Spinning fast, it has a large angular momentum, so a large torque would be needed to change the direction of its axis — even when tilted it does not fall. It is used in the guidance systems of aeroplanes, submarines and space vehicles to keep a steady course.



The gyroscope

For Your Information

When polar ice melts and the water flows towards the equator, the Earth's moment of inertia increases. To conserve angular momentum, its angular velocity decreases and the day becomes slightly longer.

Do You Know?

The flywheel called the balance wheel regulates time-keeping in mechanical clocks and watches by keeping the rate of oscillation controlled.

Point to Ponder

Why does a planet speed up when it is nearer the Sun?
$L = I\omega = mr^2\omega$ is conserved. As r decreases, ω (and the orbital speed) must increase.

Point to Ponder

Why does a coasting rotating system slow down as water drips into the beaker on it?
The added mass increases the moment of inertia, so the angular velocity decreases to keep Iω constant.

Formula Sheet

#	Formula	Description
1	$S = r\theta \Rightarrow \theta = S/r$	Relation between S, r and θ
2	$\boldsymbol{\omega}_{av} = \Delta\boldsymbol{\theta}/\Delta t$	Average angular velocity
3	$\boldsymbol{\omega} = \lim \Delta\boldsymbol{\theta}/\Delta t$	Instantaneous angular velocity
4	$\boldsymbol{\alpha}_{av} = (\boldsymbol{\omega}_f - \boldsymbol{\omega}_i)/\Delta t$	Average angular acceleration
5	$\boldsymbol{\alpha} = \lim \Delta\boldsymbol{\omega}/\Delta t$	Instantaneous angular acceleration
6	$v = r\omega \Rightarrow \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$	Relation between v, r and ω
7	$a_t = r\alpha \Rightarrow \mathbf{a}_t = \boldsymbol{\alpha} \times \mathbf{r}$	Relation between a_t, r and α
8	$\omega_f = \omega_i + \alpha t; \quad \theta = \omega_i t + \frac{1}{2}\alpha t^2; \quad 2\alpha\theta = \omega_f^2 - \omega_i^2$	Equations of angular motion
9	$a_c = v^2/r$	Centripetal acceleration
10	$F_c = mv^2/r$	Centripetal force
11	$F_c = mr\omega^2$	Centripetal force (angular form)
12	$I = mr^2$	Moment of inertia of a particle
13	$\tau = mr^2\alpha$	Torque on a particle about a fixed point
14	$\tau = I\alpha$	2nd law for rotational motion
15	$\mathbf{L} = \mathbf{r} \times \mathbf{p} \Rightarrow L = rp \sin\theta = mvr \sin\theta$	Angular momentum
16	$L = mr^2\omega = I\omega$	Angular momentum (angular form)
17	$v = \sqrt{(gR)} = \sqrt{(GM/R)}$	Critical velocity (close to Earth)
18	$T = 0, F_N = 0$	Apparent weight of a freely falling object
19	$v = \sqrt{(GM/r)}$	Orbital velocity of a satellite
20	$f = (1/2\pi)\sqrt{(g/R)}$	Spin frequency for artificial gravity
21	$I_1\omega_1 = I_2\omega_2$	Conservation of angular momentum

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PART B

Short Questions

Exercise Short Questions

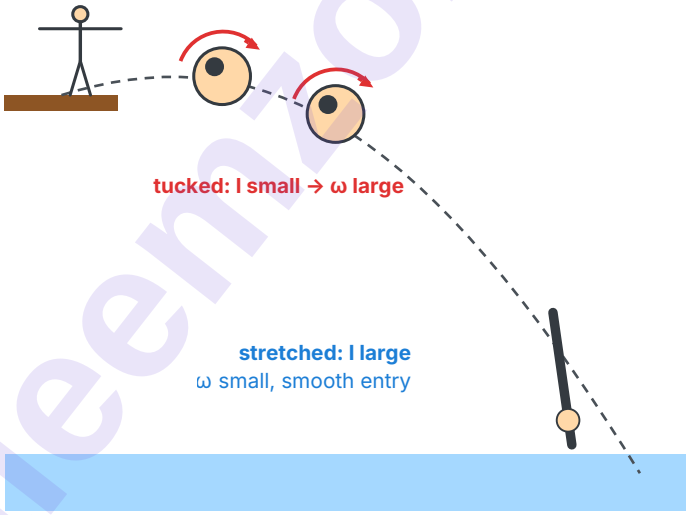
3.1 State the second law of motion in the case of rotation.

The torque acting on a body equals the rate of change of its angular momentum; equivalently, torque equals moment of inertia times angular acceleration:

$$\tau = I\alpha$$

3.2 What is the effect of changing the position of a diver while diving into the pool?

The diver changes his body position to change his moment of inertia. Taking off with arms and legs extended gives a large I and small ω. Pulling the arms and legs in reduces I, so ω increases (Iω = constant), letting him make more somersaults.



3.3 How do we get butter (cream) from milk using a centrifuge?

Milk is a mixture of light and heavy particles. When it is spun, the light particles (cream) gather near the axis of rotation while the heavy particles move outward, so cream is easily separated.

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3.4 **Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.**

Moment of inertia is the analogue. It plays the same role in rotation that mass plays in linear motion: Newton's 2nd law becomes $\tau = I\alpha$, with torque replacing force and I replacing mass. Just as mass resists changes in linear motion, I resists changes in rotation. It depends on the mass and its distribution about the axis.

3.5 **Why is it harder for a car to take a turn at higher speed than at lower speed?**

The centripetal force needed is $F_c = \frac{mv^2}{r}$. As v increases, the required force increases rapidly ($\propto v^2$). If friction cannot supply it, the car skids. So turning is harder at high speed.

3.6 **What are the benefits of using double tyres on the rear wheels of heavy vehicles?**

Double tyres increase the area in contact with the road, spreading the weight more evenly and reducing the pressure on each tyre. This improves stability, safety and load-carrying capacity.

3.7 **When a moving car turns a corner to the left, in what direction do the occupants tend to fall? Explain briefly.**

To the **right** (outward). Due to inertia, the occupants' bodies tend to keep moving in the original straight line while the car turns left, so they seem to be thrown opposite to the turn.

3.8 **Why is the acceleration of a body moving uniformly in a circle directed towards the centre?**

The direction of velocity changes continuously, and a change in velocity means acceleration. This centripetal acceleration points towards the centre, along the centripetal force, keeping the body on the circle:

$$\mathbf{a_c = \frac{v^2}{r}}$$

3.9 **How does an astronaut feel weightlessness while orbiting the Earth in a spaceship?**

Both the astronaut and the spaceship are in free fall towards the Earth under gravity. No normal reaction force acts on the astronaut, so he feels apparent weightlessness.

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SLO-Based Additional Short Questions + Past Papers (Punjab Boards)

Q1 **Define moment of inertia. Write its SI unit.**

The product of mass and the square of its distance from the axis of rotation: $I = mr^2$. SI unit kg m^2; dimensions $[ML^2]$.

Q2 **Define radian.**

The angle at the centre of a circle subtended by an arc equal in length to the radius.

Q3 **How much work is done by the centripetal force?**

Zero. The centripetal force acts along the radius while the displacement is along the tangent, at 90°:

$$W = Fd \cos 90° = Fd\,(0) = 0$$

Q4 **What are banked roads? Why are they designed so?**

Roads whose outer edge is raised above the inner edge at a turn. Banking provides the centripetal force needed by vehicles during the turn, so they do not skid.

Q5 **How is linear velocity related to angular velocity?**

For a body moving on a circle of radius r: $\mathbf{v = r\omega}$, where v is the tangential velocity and ω the angular velocity.

Q6 **What is angular displacement?**

The angle traced at the centre by the radial line of a body moving in a circle. For small values it is a vector (direction by the right-hand rule); for large values it is a scalar. SI unit: radian.

Q7 **What is torque in terms of moment of inertia?**

$F = ma$ and for circular motion $a = r\alpha$, so $F = mr\alpha$. Multiplying by r: $rF = mr^2\alpha$. With $rF = \tau$ and $mr^2 = I$:

$$\tau = I\alpha$$

Q8 **Define critical velocity and find its value.**

The velocity of a satellite orbiting very close to the Earth (height very small compared with the Earth's radius):

$$v = \sqrt{(gR)} = \sqrt{(6.4 \times 10^6 \times 9.8)} = 7.9 \times 10^3 \text{ m s}^{-1} = 7.9 \text{ km s}^{-1}$$

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PART C

Multiple Choice Questions

Exercise MCQs

1 **The ratio of the angular speed of the minute hand to the hour hand of a watch is:**

(a) 1 : 6
(b) 6 : 1
(c) 1 : 12
(d) 12 : 1

2 **A body travelling in a circle at constant speed:**

(a) has constant velocity
(b) has an inward radial acceleration
(c) is not accelerated
(d) has an outward radial acceleration

3 **A stone at the end of a long string is whirled in a vertical circle at constant speed. The tension in the string is minimum when the stone is:**

(a) at the top of the circle
(b) halfway down
(c) at the bottom of the circle
(d) anywhere in the circle

4 **Every point of a rotating rigid body has the same:**

(a) angular velocity
(b) linear velocity
(c) linear acceleration
(d) linear distance

5 **The minimum velocity necessary to put a satellite into orbit is called:**

(a) terminal velocity
(b) critical velocity
(c) artificial velocity
(d) angular velocity

6 **An astronaut is orbiting the Earth in a large capsule. Then:**

(a) he is weightless with respect to the capsule
(b) he is freely falling towards the Earth
(c) a ball projected at an angle moves in a straight line as observed by him
(d) all of the above

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7 **An object in uniform circular motion makes 10 revolutions in 2 s. Which statement is true?**

(a) its period is 2.0 s
(b) its period is 20 s
(c) its frequency is 5 Hz
(d) its frequency is 0.2 Hz

8 **A man inside an artificial satellite feels weightless because the Earth's force of attraction is:**

(a) zero at the pole
(b) balanced by the Moon's attraction
(c) equal to the centripetal force
(d) non-effective due to the satellite's design

9 **A bottle of soda is grasped by the neck and swung briskly in a vertical circle. Near which part do the bubbles collect?**

(a) near the bottom
(b) in the middle of the bottle
(c) spread throughout the bottle
(d) near the neck of the bottle

10 **The moment of inertia of a body depends upon:**

(a) mass of the body and its distribution about the axis
(b) volume of the body
(c) kinetic energy of the body
(d) angular momentum of the body

Answer Key — Exercise MCQs

1	d	2	b	3	a	4	a	5	b	6	d
7	c	8	c	9	d	10	a				

Hints

Q1: minute hand turns once per hour, hour hand once per 12 h ⇒ 12 : 1. Q3: at the top, gravity helps supply the centripetal force, so $T = mv^2/r - mg$ is least. Q7: $f = 10/2 = 5$ Hz, $T = 0.2$ s. Q9: the heavier liquid is thrown outward (to the bottom); lighter gas bubbles collect near the axis (the neck).

SLO-Based Additional MCQs + Past Papers (Punjab Boards)

1 **In two revolutions along a circle of radius r, a point covers a distance of:**

(a) $4\pi r$
(b) $3\pi r$
(c) $2\pi r$
(d) πr

2 **In one revolution the angular displacement covered is:**

(a) 60° **(b)** 90°
(c) 180° **(d)** 360°

3 **SI unit of angular displacement is:**

(a) metre **(b)** kilometre
(c) radian **(d)** centimetre

4 **If the arc length equals twice the radius, the angle traced is:**

(a) 30° **(b)** 45°
(c) 1 rad **(d)** 2 rad

5 **A wheel of radius 2 m turns through 57.3°. The tangential distance covered is:**

(a) 2 m **(b)** 4 m
(c) 57.3 m **(d)** 114.6 m

6 **π radian is equal to:**

(a) 0° **(b)** 90°
(c) 180° **(d)** 360°

7 **One radian is equal to:**

(a) 57.3° **(b)** 56.3°
(c) 360° **(d)** 58.3°

8 **2 radians =**

(a) 114.6° **(b)** 57.3°
(c) 75.3° **(d)** 37.5°

9 **Magnitude of centripetal acceleration is:**

(a) $r\omega^2$ **(b)** $r^2\omega$
(c) ω^2/r **(d)** $r^2\omega^2$

10 **The force needed to bend the straight path of a particle into a circle is:**

(a) gravitational force **(b)** frictional force
(c) centrifugal force **(d)** centripetal force

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11 **The work done by the centripetal force is:**

(a) minimum **(b)** zero
(c) maximum **(d)** negative

12 **The relation for centripetal force is:**

(a) $mr\omega$ **(b)** $mr^2\omega$
(c) $mr\omega^2$ **(d)** mvr

13 **When a body is whirled in a horizontal circle by a string, the centripetal force is supplied by:**

(a) mass of the body **(b)** velocity of the body
(c) tension in the string **(d)** air friction

14 **A 1000 kg truck turns a corner of radius 100 m at 72 km/h. The centripetal force is ___ N:**

(a) 4 **(b)** 40
(c) 400 **(d)** 4000

15 **The expression for centripetal force is:**

(a) mv^2/r^2 **(b)** m^2v^2/r
(c) m^2v^2/r^2 **(d)** $mr\omega^2$

16 **The direction of motion of a particle moving in a circle is along the:**

(a) normal **(b)** tangent
(c) away from centre **(d)** towards the centre

17 **Mud flies off the tyre of a moving bicycle along the:**

(a) tangent **(b)** normal
(c) towards the centre **(d)** perpendicular

18 **A ball rotating in a circle at the end of a string is released. It will move:**

(a) to the centre **(b)** away from the centre
(c) opposite to the motion **(d)** along the tangent

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19 **For a particle on a circle of radius r with linear acceleration a and angular acceleration α:**

(a) $a = r/\alpha$ **(b)** $a = \alpha/r$

(c) $a = r\alpha$ **(d)** $\alpha = ra$

20 **In angular motion, 2αθ is equal to:**

(a) $v_f^2 - v_i^2$ **(b)** $\omega_f - \omega_i$

(c) $v_f - v_i$ **(d)** $\omega_f^2 - \omega_i^2$

21 **The relation for torque is:**

(a) $\tau = I\alpha$ **(b)** $\tau = I/\alpha$

(c) $\tau = \alpha/I$ **(d)** $\tau = 1/(I\alpha)$

22 **A wheel 2 m in diameter makes 15 rev min^{-1}. The linear speed of a point on its rim (m s^{-1}) is:**

(a) 2π **(b)** π

(c) $\pi/2$ **(d)** $\pi/3$

23 **When a body moves in a circle, the angle between its linear velocity and angular velocity is always:**

(a) zero **(b)** 45°

(c) 90° **(d)** 120°

24 **A wheel of radius 3 m turns through 57.3°. The tangential distance is:**

(a) 3 m **(b)** 4 m

(c) 57.3 m **(d)** 114.6 m

25 **A wheel of radius 50 cm with angular speed 5 rad/s has linear speed:**

(a) 1.5 m s^{-1} **(b)** 2.5 m s^{-1}

(c) 3.5 m s^{-1} **(d)** 4.5 m s^{-1}

26 **A body rotates with ω = 2 rad s^{-1} and v = 2 m s^{-1}. The radius of the circle is:**

(a) 1 m **(b)** 0.5 m

(c) 2 m **(d)** 2.5 m

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27 **The time rate of change of angular displacement is called:**

(a) linear velocity **(b)** linear acceleration
(c) angular acceleration **(d)** angular velocity

28 **The rate of change of angular velocity is:**

(a) angular velocity **(b)** angular acceleration
(c) angular displacement **(d)** angular speed

29 **The direction of angular acceleration is always along the:**

(a) x-axis **(b)** y-axis
(c) z-axis **(d)** axis of rotation

30 **Artificial gravity is provided to a satellite by:**

(a) velocity **(b)** weight
(c) acceleration **(d)** rotating it about a vertical axis

31 **To create artificial gravity, the critical frequency of a space station about its axis is:**

(a) $(1/2\pi)\sqrt{(g/R)}$ **(b)** $(1/2\pi)\sqrt{(R/g)}$
(c) $2\pi\sqrt{(g/R)}$ **(d)** $2\pi\sqrt{(gR)}$

32 **SI unit of angular momentum is:**

(a) J s **(b)** kg m s^{-3}
(c) kg m^3 s^{-1} **(d)** kg m s^{-2}

33 **Conservation of angular momentum is important in:**

(a) diving **(b)** gymnastics
(c) ice skating **(d)** all of these

34 **In the absence of an external force, the change in momentum is:**

(a) zero **(b)** constant
(c) decreasing **(d)** increasing

35 **The critical velocity of a satellite orbiting close to the Earth is:**

(a) 5.9 km s^{-1} **(b)** 6.9 km s^{-1}
(c) 7.9 km s^{-1} **(d)** 8.9 km s^{-1}

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36 **The minimum velocity required to put a satellite into orbit is:**

(a) 5.9 km s^{-1} **(b)** 6.9 km s^{-1}

(c) 7.9 km s^{-1} **(d)** 8.9 km s^{-1}

37 **If the Earth's radius were doubled (g unchanged), the critical velocity becomes:**

(a) $v_o/\sqrt{2}$ **(b)** $v_o/2$

(c) $\sqrt{2}\, v_o$ **(d)** $v_o/4$

38 **The orbital speed of a satellite is given by:**

(a) $\sqrt{(2gR)}$ **(b)** $\sqrt{(2GM/r)}$

(c) $\sqrt{(gR)}$ **(d)** $\sqrt{(GM/r)}$

39 **Moment of inertia depends upon:**

(a) mass **(b)** choice of axis of rotation

(c) both (a) and (b) **(d)** speed of the body

40 **A mass m at distance r from a line has moment of inertia about the line:**

(a) $mr\omega$ **(b)** $mr^2\omega$

(c) $mr\omega^2$ **(d)** mr^2

41 **An angle of 30° in radians is:**

(a) $\pi/6$ **(b)** $\pi/4$

(c) $\pi/2$ **(d)** π

42 **A 10 kg body falls freely. Its apparent weight becomes:**

(a) 0 **(b)** 8.9 N

(c) 9.8 N **(d)** 10 N

43 **2 revolutions are equal to:**

(a) π rad **(b)** 2π rad

(c) $3\pi/2$ rad **(d)** 4π rad

44 **The dimension of angular velocity is:**

(a) $[LT^{-1}]$ **(b)** $[LT^{-2}]$

(c) $[L^{-1}T]$ **(d)** $[T^{-1}]$

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45 **The rotational analogue of force is:**

(a) velocity **(b)** torque

(c) mass and weight **(d)** momentum

46 **The relation between torque and angular acceleration is:**

(a) $\tau = I\alpha$ **(b)** $\tau = I/\alpha$

(c) $\tau = \alpha/I$ **(d)** $\tau = 1/(I\alpha)$

Answer Key — Additional MCQs

1	a	2	d	3	c	4	d	5	a	6	c
7	a	8	a	9	a	10	d	11	b	12	c
13	c	14	d	15	d	16	b	17	a	18	d
19	c	20	d	21	a	22	c	23	c	24	a
25	b	26	a	27	d	28	b	29	d	30	d
31	a	32	a	33	d	34	a	35	c	36	c
37	c	38	d	39	c	40	d	41	a	42	a
43	d	44	d	45	b	46	a				

Hints

Q4: $\theta = S/r = 2r/r = 2$ rad. Q5: $57.3° = 1$ rad $\Rightarrow S = 2 \times 1 = 2$ m. Q14: $v = 72$ km/h $= 20$ m/s $\Rightarrow F = 1000 \times 400/100 = 4000$ N. Q22: $r = 1$ m, $\omega = 15 \times 2\pi/60 = \pi/2$ rad/s $\Rightarrow v = \pi/2$. Q25: $0.5 \times 5 = 2.5$ m/s. Q26: $r = v/\omega = 1$ m. Q37: $v = \sqrt{(gR)}$, so doubling R gives $\sqrt{2}\, v_o$.

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PART D

Solved Exercise

Exercise Numericals

3.1 **A laser beam is directed from the Earth to the Moon. The beam spreads over a diameter of 2.50 m at the Moon's surface. What is the divergence angle of the beam? The distance of the Moon from the Earth is 3.8×10^8 m.**

Given: S = 2.50 m • r = 3.8×10^8 m • θ = ?

$$S = r\theta \Rightarrow \theta = \frac{S}{r} = \frac{2.50}{3.8 \times 10^8}$$

$$\boldsymbol{\theta \approx 6.58 \times 10^{-9} \text{ rad}}$$

3.2 **A car moves at 108 km h^{-1}. If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1}.**

Given: v = 108 km h^{-1} = 108 × 1000/3600 = 30 m s^{-1} • D = 0.6 m ⇒ r = 0.3 m

$$\boldsymbol{\omega = \frac{v}{r} = \frac{30}{0.3} = 100 \text{ rad s}^{-1}}$$

$$\boldsymbol{\omega = \frac{100}{2\pi} \text{ rev s}^{-1} = \frac{100}{2 \times 3.14} \approx 15.92 \text{ rev s}^{-1}}$$

3.3 **An electric motor runs at 1800 rev min^{-1}. On switching off, it comes to rest in 20 s. If the retardation is uniform, find the number of revolutions it makes before stopping.**

Given: ω_i = 1800 rev min^{-1} = 30 rev s^{-1} • $\omega_f = 0$ • t = 20 s

$$\omega_{av} = \frac{\omega_i + \omega_f}{2} = \frac{30 + 0}{2} = 15 \text{ rev s}^{-1}$$

$$\boldsymbol{\theta = \omega_{av} \times t = 15 \times 20 = 300 \text{ revolutions}}$$

3.4 **A string 0.5 m long holding a stone can withstand a maximum tension of 35.6 N. Find the maximum speed at which a 0.5 kg stone can be whirled with it in a vertical circle.**

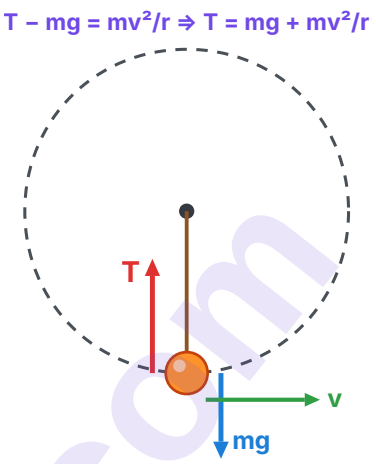
Given: r = 0.5 m • T_{max} = 35.6 N • m = 0.5 kg • v = ?

Tension is greatest at the **bottom** of the circle:

$$T = mg + \frac{mv^2}{r}$$

$$35.6 = 0.5 \times 9.8 + \frac{0.5\,v^2}{0.5} \Rightarrow 35.6 = 4.9 + v^2$$

$v^2 = 30.7 \Rightarrow v \approx 5.54$ m s^{-1}



Forces at the bottom of the circle

3.5 **The flywheel of an engine rotates at 2100 rev min^{-1} when the power is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m^2.**

Given: ω_i = 2100 rev min^{-1} = 35 rev s^{-1} = 70π rad s^{-1} • ω_f = 0 • t = 3 min = 180 s • I = 36 kg m^2

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 70\pi}{180} = -1.22 \text{ rad s}^{-2}$$

$$\tau = I\alpha = 36 \times \left(\frac{-70\pi}{180}\right) \approx -43.98 \text{ N m}$$

$|\tau| \approx 44$ N m (negative sign: opposes rotation)

3.6 **What is the moment of inertia of a 200 kg sphere whose diameter is 60 cm?**

Given: m = 200 kg • D = 0.6 m ⇒ r = 0.3 m

For a solid sphere about its diameter, I = ⅖ mr^2:

$$I = \tfrac{2}{5} \times 200 \times (0.3)^2 = 80 \times 0.09$$

I = 7.2 kg m^2

3.7 **A satellite orbits the Earth at an altitude of 200 km. Taking the Earth's radius as 6400 km, calculate the orbital speed of the satellite.**

Given: h = 200 km = 2×10^5 m • R = 6.4×10^6 m • r = R + h = 6.6×10^6 m • G = 6.67×10^{-11} N m^2 kg^{-2} • M = 6×10^{24} kg

$$v = \sqrt{\left(\frac{GM}{r}\right)} = \sqrt{\left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.6 \times 10^6}\right)}$$

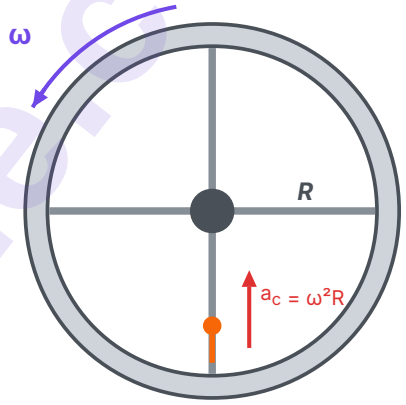
$v \approx 7.79 \times 10^3$ m s^{-1} $\approx$ 7.8 km s^{-1}

3.8 **A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s^{-1}. What artificial gravity do the astronauts experience?**

Given: r = 20 m • ω = 0.5 rad s^{-1}

$$g' = r\omega^2 = 20 \times (0.5)^2 = 20 \times 0.25$$

$g' = 5$ m s^{-2}



Spinning space station

3.9 **A bicycle wheel has an angular momentum of 10 kg m^2 s^{-1} and angular velocity of 2 rad s^{-1}. Find its moment of inertia.**

Given: L = 10 kg m^2 s^{-1} • ω = 2 rad s^{-1}

$$L = I\omega \Rightarrow I = \frac{L}{\omega} = \frac{10}{2}$$

$I = 5$ kg m^2

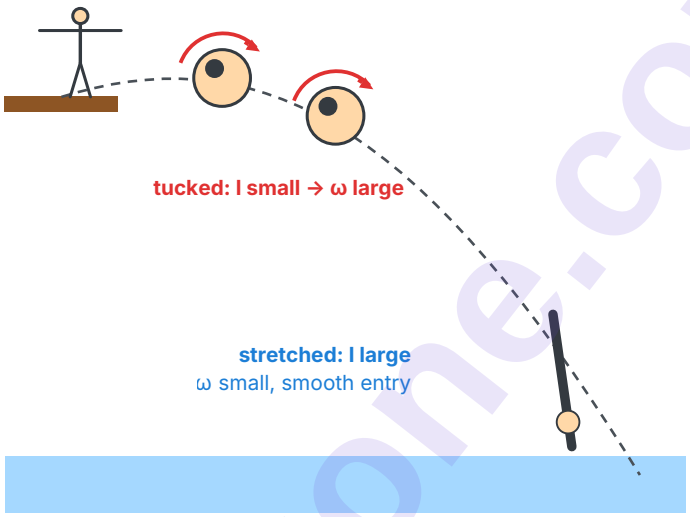
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3.10 **A diver comes off a board with arms straight up and legs straight down, giving a moment of inertia of 18 kg m^2. He then tucks into a ball, reducing it to 3.6 kg m^2. Tucked, he makes two complete rotations in 1.0 s. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water?**

Given: $I_1 = 18$ kg m^2 • $I_2 = 3.6$ kg m^2 • $\omega_2 = 2$ rev s^{-1} • $t = 1.5$ s

$$I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_1 = \frac{I_2}{I_1}\omega_2 = \frac{3.6}{18} \times 2 = 0.4 \text{ rev s}^{-1}$$

$\theta = \omega_1 t = 0.4 \times 1.5 = 0.6$ revolution



Constructed Response Questions

3.1 **If the angular velocity of different particles of a rigid body is constant, will their linear velocities also be constant (the same)?**

No. Since v = rω, linear velocity depends on the distance r from the axis. Particles at different distances have different linear velocities even with the same ω.

3.2 **A loaf of bread is lying on a rotating plate. A crow takes away the loaf and the plate's rotation increases. Why?**

Removing the loaf reduces the mass on the plate, so its moment of inertia I decreases. By conservation of angular momentum (L = Iω = constant), ω increases.

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3.3 Why do we tumble when we take a sharp turn at large speed?

1. Inertia: moving in a straight line, our body tends to keep doing so (Newton's 1st law).
2. Centripetal force: a sharp turn needs $F_c = mv^2/r$ to change our direction. At large speed, if this force is insufficient, we tumble outward and lose balance. To avoid tumbling we slow down or lean inward.

3.4 What will be the time period of a simple pendulum in an artificial satellite?

$T = 2\pi\sqrt{(\ell/g)}$. In the satellite $g_{eff} \approx 0$ (weightlessness), so $T \to$ **infinity**: the pendulum does not oscillate.

3.5 Is the motion of a satellite in its orbit uniform or accelerated?

Accelerated. Its speed may be constant, but its direction changes continuously due to gravity towards the Earth's centre. A change in direction means the velocity changes, giving a centripetal acceleration $a_c = v^2/r$. So uniform circular motion is accelerated motion.

3.6 What are the advantages of the radian being preferred as the SI unit over the degree?

1. Simpler formulas: many formulas are simpler in radians ($S = r\theta$, $v = r\omega$, $a = r\alpha$).
2. Unitless: the radian is a ratio of two lengths (dimensionless), so it is easier to use in equations.

3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.

In one revolution the body returns to its starting point, so total displacement = 0:

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{0}{\Delta t} = 0$$

Initial and final velocities are equal (same speed, same direction), so the change in velocity is zero:

$$\textbf{average acceleration} = \frac{\textbf{change in velocity}}{\textbf{total time}} = \frac{\mathbf{0}}{\mathbf{\Delta t}} = \mathbf{0}$$

3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?

The direction of the rain **relative to you**. Tilt the umbrella towards the wind, along the slanting rain, so it blocks the rain coming in at an angle.

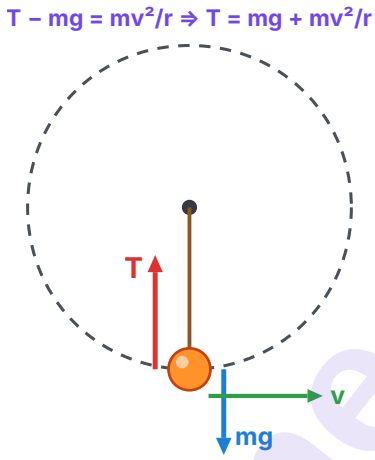
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3.9 **A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain why.**

At rest the tension only balances the weight: T = mg. In a vertical circle, at the bottom the string must supply the weight **and** the centripetal force:

$$T = mg + \frac{mv^2}{r}$$

This is more than the string can bear, so it breaks.



3.10 **How is the centripetal force supplied in the following cases: (i) a satellite orbiting the Earth (ii) a car taking a turn on a level road (iii) a stone whirled in a circle by a string?**

(i) **Gravitational force**: $F = GMm_s/r^2$.
(ii) **Friction** between the tyres and the road.
(iii) **Tension** in the string: $T = mv^2/r$.

Comprehensive Questions

3.1 **What is meant by angular momentum? Explain the law of conservation of angular momentum with daily-life examples.**

See **Long Question Q11 and Q12**.

3.2 **Show that orbital angular momentum L = Iω.**

See **Long Question Q11**.

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3.3 **Define moment of inertia. Prove that the torque on a rotating rigid body equals the product of its moment of inertia and angular acceleration.**

See **Long Question Q10**.

3.4 **What are artificial satellites? Calculate the minimum time period needed to put a satellite into orbit.**

See **Long Question Q6**.

3.5 **Define orbital velocity. Derive an expression for it.**

See **Long Question Q7**.

3.6 **Write a note on artificial gravity. Derive an expression for the frequency with which a spaceship rotates to provide artificial gravity.**

See **Long Question Q9**.

3.7 **Prove that (i) $v = r\omega$ and (ii) $a = r\alpha$.**

See **Long Question Q2 and Q3**.

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